Research Article

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Analysis of Students' Reasoning when Solving an Algebraic Generalisation Activity

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Abstract

The aim of this study is to analyse how Moroccan pupils in the sixth year of primary school (11-12 years old) and the first year of secondary school (12-13 years old) use their reasoning and symbols to solve a task based on figurative regularities. The proposed activity was structured and analysed according to Dörfler's model (1991), integrating reasoning processes based on this model. The results of this analysis were used to identify students' predispositions and potential for solving this type of task, as well as the different types of symbolisms used before and after their introduction to algebra.

Keywords: *Reasoning, algebraic thinking, generalisation, symbolisation, figurative pattern*

1. Introduction

The question of the arithmetic-algebraic transition has become a major concern for educational researchers and mathematics teachers. Indeed, many students encounter difficulties in their first approach to algebra in lower secondary school, leading to a lack of understanding of concepts, a tendency to memorise rules without grasping their meaning, and a resistance to adopting a more abstract and generalised approach (Bednarz et al., 1996; Jeannotte, 2005; Kieran, 1991). Faced with these challenges, many researchers (Booth, 1984; Kieran, 1992) have attempted to understand the cognitive mechanisms underlying this transition and to identify the factors influencing students' understanding of algebra. In this pursuit, work such as that of Booth (1984) has highlighted the issues associated with the epistemological break between arithmetical and algebraic thinking. This epistemological break manifests itself in notable differences in problem-solving strategies, the interpretation of algebraic and numerical notations, as well as the perception of the status of the equality sign and the use of letters in algebra. Similarly, Kieran (1992) has pointed out that students often tend to resort to memorising rules and procedures in algebra, seeing these activities as the very essence of the discipline, without grasping the underlying deeper concepts. Some researchers, such as Chevallard (1989a, 1989b, 1989-1990), have even described this transition as an epistemological break, leading to a radical transformation of the thought process. In response to these problems, the 'Early Algebra' movement emerged at the end of the 1990s, advocating the early introduction of algebraic thinking from primary school onwards, without using the formal language of algebra. The aim of this approach is to provide pupils with opportunities to develop their algebraic thinking from the earliest stages, deepening their understanding of fundamental mathematical concepts such as operations, equations, formulae, variables and variations (Kaput, 1998; Squalli, Mary, Marchand, 2011). Larguier (2015) conducted a study of early encounters with algebra, revealing indicators of students' development of algebraic thinking. Pupils demonstrate algebraic thinking by recognising patterns, formulating generalisations using examples, solving unknowns when solving problems, detaching from specific contexts and validating general terms with known numerical data.

Studies of Moroccan pupils (Abouhanifa et al., 2018; Abouhanifa, 2019; Moukhliss et al., 2022; Moukhliss et al., 2023; Ennassiri et al., 2022) have shown that at middle school, pupils have difficulty mobilising algebraic tools in problems. The way in which students approach generalisation in activities clarifies the elements that hinder the development of algebraic thinking, the methods they use to generalise sequences of patterns.

In this study, we seek to answer the following research questions:

1. What reasoning do Moroccan primary school pupils adopt when faced with generalisation activities based on figurative motifs?
2. What symbols do they use to express this generalisation?

Based on the difficulties associated with the transition to algebra, this study seeks to modify the way pupils' approach algebraic concepts, with implications for teaching methods and curricula. The study also seeks to highlight the importance of promoting algebraic thinking from an early age and suggests ways of enriching primary school mathematics curricula by incorporating activities likely to develop algebraic reasoning skills.

2. Generalisation Activities as a Means of Developing Algebraic Thinking

2.1 How do they encourage the development of algebraic thinking?

In the context of the development of algebraic thinking, Radford (2014) and Kieran et al. (2016) have described four crucial foundational elements of algebraic thinking: (1) generalisation related to numerical and geometric models, (2) generalisation concerning the properties of numerical operations and structures, (3) the representation of relations between quantities, and (4) the introduction of alphanumeric notation. Furthermore, Radford (2014) characterises algebraic thinking through three distinctive features:

Indeterminacy: This characteristic denotes the presence of unspecified quantities, such as unknowns, variables, parameters, etc.

Denotation: Denotation allows these indeterminate quantities to be represented using symbols, gestures or other symbolic means.

Analyticity: The concept of analyticity allows unknowns to be manipulated as if they were known, enabling various operations to be performed on them.

A generalisation activity can be defined as an exercise that requires the implementation of a generalisation process in order to discover a model or a rule (Squalli et al., 2020, p. 46). Various studies have highlighted the importance of this type of activity in the development of algebraic thinking in pupils from primary school onwards (Larguier, 2015; Kieran et al., 2016; Radford, 2014). According to several authors (Mason, 1996; Radford 2008, 2014), this activity is considered a favourable environment for facilitating the transition between arithmetic and algebra by developing algebraic thinking, as it encourages reflection aimed at identifying and expressing regularities (Vlassis et al., 2017). Once a student has successfully generalised the process and formulated a rule, they engage in reasoning that involves the manipulation of indeterminate quantities, thus constituting an essential element in the development of algebraic thinking (Radford, 2014). These activities provide an environment conducive to the development of algebraic thinking for two main reasons:Firstly, pupils are encouraged to reason analytically when manipulating unknown quantities. Secondly, the diversity of approaches possible in generalisation activities is a real asset and provides an opportunity to tackle many mathematical concepts, such as the notion of equivalence of expressions. In addition, Radford (2014) emphasises the significant relationship between spatial and numerical structures, which must be considered in order to promote algebraic thinking. This relationship allows connections to be made between the quantity of a shape and its applicable parts. When students decompose numbers, they create associations between known and unknown entities, which allows them to perform calculations (Callejo and Zapatera, 2017, p.313).

2.2 Students' reasoning in generalising from the context of arithmetic sequences.

Radford (2006, 2008) has identified three types of reasoning used by students when generalising from the context of arithmetic sequences. We summarise them briefly here.

Naive Induction: Students tend to rely on the analysis of a single familiar pattern to deduce a rule that could be applied to the calculation of an unknown pattern. This approach frequently involves proportional reasoning or the use of a rule of three. However, this method is often incorrect because it neglects consideration of all the available patterns.

Arithmetic generalisation: Students observe a steady increase between two successive patterns in a sequence. This approach allows them to make predictions about similar patterns, but does not facilitate generalisation. As a result, they may find it difficult to quickly identify distant terms in the sequence. This type of reasoning is limited to numerical values and does not involve the use of operations with indeterminate quantities, which is essential for developing algebraic thinking. This type of generalisation is often encountered when examining arithmetic sequences where the figurative patterns are presented in ascending order. To help students make the transition from arithmetic to algebraic generalisation, it is essential that they become aware of the number of iterations required to add the necessary number of elements when moving from one pattern to another (Fagnant and Demonty, 2020). They should be encouraged to think multiplicatively rather than additively to achieve algebraic generalisation. In this way, they can develop a deeper understanding of patterns and sequences.

Algebraic generalisation: In this third mode of reasoning, students identify a regular pattern based on the first known patterns, which they can then generalise to determine any term in the sequence. In this way, they establish a formula that depends on the number of the pattern and the number of elements in it.

As far as the semiotic aspect of generalisation is concerned, students tend to use annotations, calculations or sentences when they adopt one of the first two approaches (naive induction or arithmetic generalisation). However, when it comes to algebraic generalisation, it can manifest itself in a variety of ways. Radford (2006) has identified three types of symbolisation to represent this indeterminate quantity.

Factual algebraic generalisation: The students succeed in identifying a regularity, but this is only presented using numerical models. They represent the unknown element using a numerical value.

Contextual algebraic generalisation: Students use a substitution symbol to represent an indeterminate quantity, which may be a question mark, three suspension points, an empty case or any other representation. This symbolisation may involve the use of letters, but it remains correlated with the context. In this illustration, we could imagine a generalisation such as "the pattern number multiplied by 3 plus 2," or nx3+ 2 Demonty and Fagnant, 2018).

Symbolic algebraic generalisation: Students use the symbolic language of algebra to derive a general formula that is entirely valid mathematically, regardless of the context of the sequence. In this scenario, they become a formula of the form 3n + 2 = c. Such a generalisation is only possible after being introduced to the teaching of formal algebra.

2.3 The development of the generalisation process: Dörfler's model (1991)

Squalli (2015) develops the analytical framework proposed by Dörfler (1991) with an objective distinct from that of Radford (2008). Rather than simply observing reasoning or modes of expression, Squalli's (2015) approach aims to describe the 'ideal' development of the algebraic generalisation process itself. This model provides a better understanding of the structuring of the various possible stages that are essential to progress towards algebraic generalisation, as proposed by Radford (2008). It also provides an analysis of how students progress through the generalisation process. Figure 1, as presented by Squalli (2015), illustrates the key stages of this process as follows:

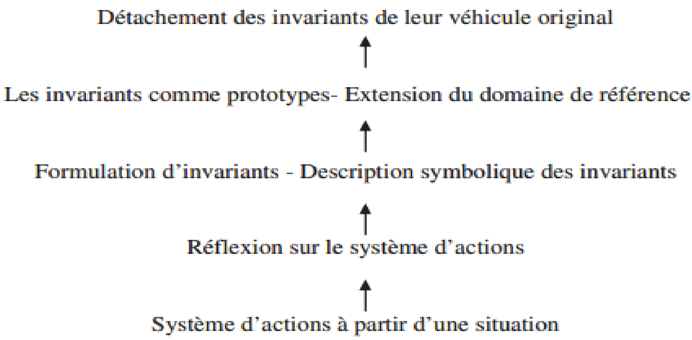


Figure 1: Simplified generalisation model of Dörfler (1991) proposed by Squalli (2015)

According to the model developed by Squalli (2015), the generalisation process begins with an "action" or a "system of actions from a situation", which directs the subject's attention towards specific relationships between the objects involved in these actions. Repetition of these actions gradually leads the subject to identify certain regularities or essential 'invariants' of the actions, leading to a symbolic formulation, whether verbal, iconic, geometric or algebraic. A crucial moment in this process occurs when these invariants are replaced by 'prototypes', where symbolic formulations are no longer limited to describing specific cases, but extend to potential cases. Subsequently, the invariants and their symbolic formulations are dissociated from the original actions. From then on, the symbols acquire a specific domain of reference, which is extended more rapidly. They thus become autonomous mathematical objects, whose meaning is based on the invariants. Complete detachment from the original actions leads to the production of a correct formula, which is no longer linked to contextual elements.

3. Research Methodology

3.1 Data collection

Data collection began with a generalisation problem based on figurative motifs, with three separate questions. Students were asked to solve these problems and provide detailed written answers in their worksheets. Analysis of the quantitative data aimed to identify patterns of reasoning and general trends in the students' responses. The most frequently used solving strategies were examined, as well as the different methods by which students expressed algebraic reasoning to represent mathematical generalisations. This analysis highlighted how students conceive and translate algebraic models related to generalisation.

3.2 Study group

The study involved a total of 156 pupils, 74 in primary school and 82 in secondary school. The primary pupils were in Year 6, while the secondary pupils were in Year 1, belonging to 6 different educational establishments. The 6th graders had no prior exposure to algebra, while the 1st graders had already been introduced to algebra through a course entitled "Equations", followed by chapters on the development and factoring of algebraic expressions. Each student was individually presented with a test consisting of a three-question problem, without any intervention from the teacher, and it was stressed that participation in the study was voluntary.

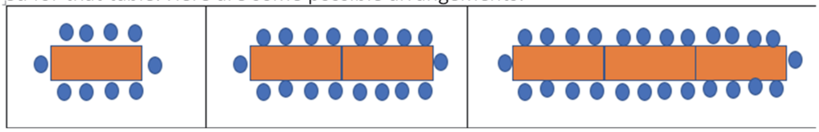
3.3 Conditions for administering the questionnaire to pupils

The aim of this study was to observe how pupils interacted with generalisation and symbolisation when they took part in an activity linked to an arithmetic sequence. The activity involved the use of small tables with chairs and was a novelty for the students, who had never before been confronted with this type of pattern-based task. The students worked individually on the activity for 60 minutes without direct intervention from the teacher, except to clarify instructions if necessary. Teachers were allowed to intervene only to clarify the task. All participants, teachers and school leaders, were informed that their participation in the study was voluntary.

3.4 Prior analysis of activity

Figure 2 shows the original activity proposed to the students. This activity was developed in several stages, with successive trials and adjustments. Its primary aim was to encourage an algebraic approach in the pupils by getting them to think about indeterminacy and to use variables to represent unknown quantities. In addition, particular attention was paid to the diversity of numerical and algebraic expressions generated by the activity, while following the generalisation model proposed by Dörfler (1991) and Squalli (2015) (Figure 1).

Amine is a furniture designer. He has small rectangular tables that he would like to assemble to create large tables, so that he can offer his customers customised tables to suit their needs. Each small table can be combined with others to form a large table. Each small table is accompanied by a specific number of chairs required for that table. Here are some possible arrangements:



1. How many chairs will be needed to assemble a large table using 2 small tables? Using 3 small tables? Using 10 small tables? Using 107 small tables?
2. Amine wants to work out the total number of chairs needed to make a large table, whatever its length. You can help him by explaining how to calculate the number of chairs needed based on the number of small tables used in the design of the furniture.
3. Find a mathematical expression that calculates the total number of chairs needed to form a large table, whatever its length, using small tables in Amine's furniture design.

Figure 2: Generalisation activity based on figurative patterns used in the context of our study

In the first part of the activity, the pupils were asked to count the number of chairs using 2, 3 and 10 tables. However, the aim was not so much to get them to do a rigorous count as to encourage them to think about the reasoning to be applied. They were then given a calculation problem, which consisted of determining the number of chairs needed for 107 small tables. The aim of this question was to encourage the students to apply the same process as before, but this time to a specific example, focusing on the pattern of the calculation rather than the details of the calculation itself. This required them to think more abstractly, as concrete counting was not possible. This led them to generalise the rules and models from previous cases and apply them to the new case with relevant invariants. With the aim of developing the notion of indeterminate number, question 2 was formulated in such a way as to encourage the students to anticipate unspecified situations. They were asked to explain how to calculate the number of chairs needed according to the number of small tables used in the design of the furniture. The aim of this approach was to transform the invariants observed in the specific cases into more general prototypes, thereby facilitating the emergence of generality. The students were free to express this generality in different languages, whether in ordinary terms, using mathematical symbols or adopting a formal formulation. Finally, in question 3, the students were encouraged to express explicitly and mathematically the variable representing the generality established in the previous question. This led them to separate the symbolic formulation from the initial process, enabling them to apply syntactic mathematical manipulations consistent with the rules of calculation, independently of the specific meanings attached to the real objects. This crucial step strengthened their understanding of algebraic concepts and prepared them to solve more complex problems requiring an abstract and symbolic approach.

4. Results

4.1 Breakdown of successful and unsuccessful production for each task.

The results are presented along two main lines, corresponding to the two research questions set out above. The first focuses on the processes of generalisation, by examining the different patterns of action and invariants created by the pupils. The second concerns the study of the symbolisations produced by the pupils.

We begin by presenting the breakdown of the productions analysed according to their success or failure, as illustrated in Table 1.

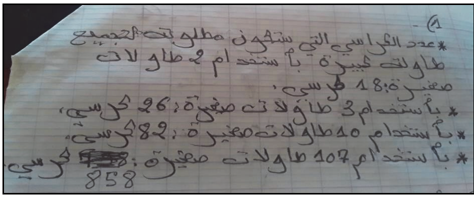
Table 1: Distribution of students according to task success

		Primary		Secondary	
		Successful	*Unsuccessful*	*Successful*	*Unsuccessful*
Question 1	*Task 1*	83.78% (62)	16.22% (12)	97.56% (80)	2.44% (2)
	Task 2	81.08% (60)	18.92% (14)	93.85% (77)	6.15% (5)
	Task 3	59.46% (44)	40.54% (30)	60.98% (50)	39.02% (32)
	Task 4	37.8% (28)	62.2% (46)	42.68% (35)	57.32% (47)
Question 2		12.16% (9)	87.84% (65)	21.95% (18)	78.05% (64)
Question 3		24.32% (18)	75.68% (56)	32.87% (27)	67.13% (55)
		100% (74)		100% (82)	

Table 1 shows that over 80% of primary school pupils and over 90% of first-year secondary school pupils passed the first two tasks in the first question.

However, the success rate fell in the next two tasks, and even more so in the last two questions. Specifically, only 24.32% of primary school pupils succeeded in the third question, and only 32.87% of secondary school pupils succeeded in this task. This implies that pupils have difficulty anticipating unspecified situations and are unable to transform the invariants observed in specific cases into more general prototypes, which hampers the emergence of generality.

Some of the students' correct answers simply gave the number of chairs without justification (see Figure 3).



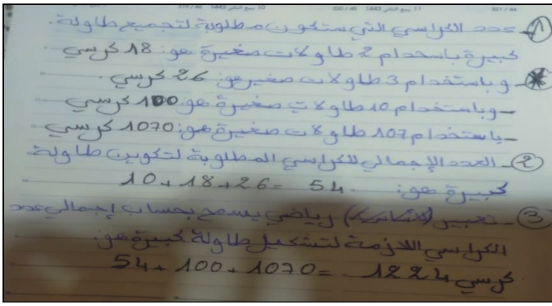
Translation:

1. Number of chairs needed to assemble a large table using 2 small tables: 18 chairs.
2. Using 3 small tables: 26 chairs.
3. Using 10 small tables: 82 chairs.
4. Using 107 small tables: 858 chairs.

Figure 3: An example of the reasoning of student 1 who did not justify his answers."

Other students also gave incorrect answers without justification (see Figure 4).

Pupil 2 was able to find the number of chairs for 2 tables and 3 tables by counting from the diagram shown. However, when he tried to calculate the number of chairs for 10 tables and 107 tables, he multiplied the number by 10 based on the first pattern showing the number of chairs for one table. These incorrect answers appear to be the result of naive inductive reasoning (Radford, 2008) which led to erroneous generalisations. This student may not have understood what was being asked of him when solving the second question. He did not understand that the aim was to provide a method for calculating potential cases. He relied solely on the visualised diagrams, adding together the chairs represented in the first problem figure with those in the second and third figures. We think that this student lacks the abstract character and the ability to link ideas and concepts logically.

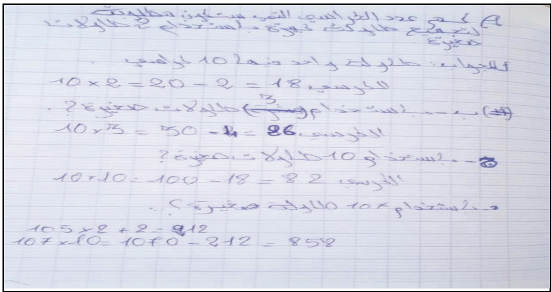


Translation:

1. Number of chairs needed to assemble a large table using 2 small tables: 18 chairs.
   Using 3 small tables: 26 chairs.
   Using 10 small tables: 100 chairs.
   Using 107 small tables: 1070 chairs.
2. The total number of chairs needed to assemble a large table is: 10 + 18 + 26 = 54.
3. Mathematical expression to calculate the number of chairs needed to assemble a large table: 54 + 100 + 1070 = 1224 chairs.

Figure 4: Student 2's response

Another type of result was discovered (see Figure 5). This student clearly understood the process of determining the number of chairs by multiplying the number of tables by 10, then subtracting the number of chairs in the middle. The number of chairs in the middle is calculated on the basis of two chairs for each middle table and one chair for the side tables. All this leads to the formula 10n - (n-1) x2 However, the student did not take into account the structural aspect of the formula, focusing mainly on the procedural aspect. Instead of seeing equality as an equivalence relationship, he saw it as a simple announcement of a result. He took the approach of multiplying the number of chairs in the same line by 10, then subtracting 1 from the number of chairs, and finally multiplying by 2. This approach did not allow him to fully grasp the overall meaning of the formula.



Translation:

1. How many chairs will you need to assemble a large table using 2 small tables?

A- Answer: A table with 10 chairs. 10 x 2 = 20 - 2 = 18 chairs.

B- Using 3 small tables

10 x 3 = 30 - 4 = 26 chairs.

C- Using 10 small tables

10 x 10 = 100 - 18 = 82 chairs.

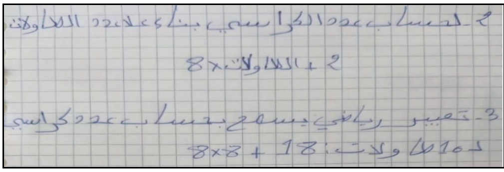
D- Using 107 small tables 105 x 2 + 2 = 212.

107 x 10 = 1070 - 212 = 858.

Figure 5: An example of the reasoning of student 3, who did not take into account the equivalence aspect of equality

For a better understanding and appropriation of the formula, the student should have interpreted the equality as a correspondence between two expressions: on the one hand, the total number of chairs represented by 10n (where n is the number of tables), and on the other hand, the number of chairs taking into account the two chairs for each middle table and one chair for the side tables, expressed as (n-1) x 2. By establishing this equality, he could have concluded that 10n - (n-1) x 2 is the method for calculating the total number of chairs. By understanding the structure of the formula, the student could have generalised his reasoning and applied it to other similar situations, which would have strengthened his mathematical understanding. It is therefore essential to make students aware of the conceptual aspect of formulas and the meaning of equations, so that they can solve problems more thoroughly and creatively.

We note that there are some students (Figure 6) who demonstrate their ability to manipulate invariants and establish relevant mathematical equivalences. This is evidence of in-depth reflection on action systems.



Translation:

2. To calculate the number of chairs from the number of tables: 8 x Tables + 2
3. Mathematical formula for calculating the number of chairs for 10 tables : 8 x 8 + 18

Figure 6: An example of the reasoning of pupil 4, who is in the primary class.

In this answer (Figure 6), two types of generalisation are presented. The first generalisation, proposed in question 2, is a contextual generalisation, while the second, set out in question 3, is a factual generalisation. Through these generalisations, the students attempt to demonstrate the generality of the subject matter.

The student has provided two different formulae for the action system. The first formula is expressed as 8n+2, with invariants 8 and 2. The second formula is 8(n-2) +18, with invariants 8 and 18. Note that the operations of multiplication and addition are invariants common to both formulae. This activity generates equivalent numerical and/or algebraic expressions. These different expressions open up the possibility of tackling the meaning of equivalence and confirming the results of Samson (2012).

4.2 *Initial action systems and corresponding invariants*

Analysis of the pupils' responses revealed the emergence of different systems of action, five of which led to correct generalisations, as explained in Table 2 below. Table 2 shows the distribution of pupils whose action systems led to correct answers: 29.7% (22) of primary school pupils and 39% (32) of secondary school pupils.

Table 2. Description of action systems

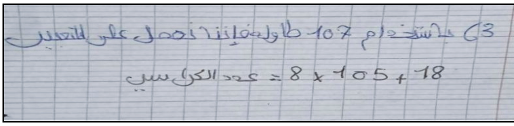
Action systems	Description	Literal formula	Number of responses (Primary)	Number of responses (secondary)
A	Take eight times the number of tables, then add two chairs.	8n+2	63.64% (14)	37.5 % (12)
B	Take ten times the number of tables and subtract the chairs shared between neighbouring tables	10n-2x(n-1)	4.55% (1)	21.88% (7)
C	Take the first table and the last table, both with 9 chairs each. Then add the middle tables, each containing 8 chairs.	8x (n-2) +18	27.27% (6)	28.13 % (9)
D	Take the four chairs on the left and right, multiply them by the number of tables, then add the chairs on either side.	4n+4n+2	4.55% (1)	9.38 % (3)
E	Take the total number of tables and multiply it by the initial number of chairs, which is 10 for the initial table. Next, subtract the number of chairs from the middle tables and remove two chairs from each side table.	10t - 2m – 2	0% (0)	3.13% (1)

Table 2 shows the varied range of methods adopted by the students, mainly in the first year of secondary school, although families A and C were mainly favoured. A total of five distinct approaches were used by students at both levels. This diversity in the methods used by the students demonstrates the potential of this type of activity to foster the development of algebraic skills. Indeed, the comparative examination of these various formulae or expressions will help them to reconsider the meaning of arithmetical operations, as well as concepts such as equality, variables and techniques such as the simplification of similar terms, distributivity and even particular products.

4.3 Different types of generalisation

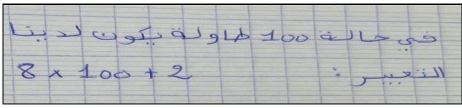
Symbolic generalisation was only used by pupils in the 1st year of secondary school, while no primary school pupils used this type of generalisation. Most of the primary pupils opted for factual generalisation, whereas this type of generalisation was rarely found among the secondary pupils. Furthermore, we note that 5 primary and 10 secondary pupils opted for contextual generalisation. These results concern only those students who succeeded in identifying a system of action.

Factual generalisation: Figures 7 and 8 show the productions of student 5 and student 6 concerning generalisation. Student 5 adopts a generalisation of the type 8(n-1) +18, while student 6 proposes a generalisation of the type 8n+2.



Translation:
Using 107 tables, we obtain the expression Number of chairs=8x105+18

Figure 7: An example of the reasoning of student 5, who provided a factual generalisation.

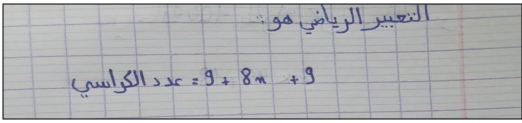


Translation:
In the case of 100 tables, we have the expression: 8x100+2

Figure 8: An example of the reasoning of student 6, who provided a factual generalisation.

The first example illustrates the spontaneous generalisation made by student 1. It can be seen that both the general expression in words and that in mathematical language refer to a single example. However, the use of the terms " **using... will have** " before these two expressions indicates the general nature of this numerical case, which is used to explain a general approach. The adverb "**in the case of**", used by student 2, expresses the generality of an operation that applies continuously to any number of tables, and this term plays the role of a generative function of the language (Radford, 2000).

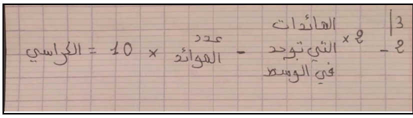
Contextual algebraic generalisation: The formulas developed by the students as part of their contextual generalisation reflect all the steps taken by the students to arrive at a generalisation. Indeterminates are expressed using generic terms directly linked to the spatio- temporal situation. In pupil 7's answer (Figure 9), the pupil established the expression to indicate the order of the operations carried out, without it being necessary to do so, by respecting the rule of priority of the operations, which gave rise to the elaborated action system (9 + 8n + 9).



Translation:
The mathematical expression is: The number of chairs=9+8n+9

Figure 9: An example of the reasoning of student 7 who provided a contextual generalisation

However, this generality remains tied to the original situation by maintaining the hierarchy of operations (starting with adding 9, then multiplying n by 8, then adding 9).

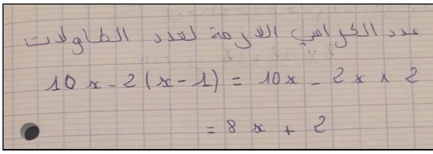


Translation:
Number of chairs = 10 x number of tables- Tables in the middle x 2 -2

Figure 10: An example of the reasoning of student 8 who provided a contextual generalisation

In the response from pupil 8 (Figure 10), the pupil uses three indeterminates: "the number of chairs", "the number of total tables" and "the number of tables in the middle", giving rise to the elaborated action system (c = 10t - 2m - 2). This action system reveals an understanding that remains attached to the context, whose symbolisation retains traces of the situations that gave rise to it. In this case, the student expressed the indeterminates using generic words directly linked to the spatiotemporal situation rather than nouns. However, the student did not grasp the possibility of representing the two indeterminates that represent the same object, the table, by the same noun. These two problems reveal two relevant concepts as determined by Valasy et al. (2018). The first concept is that of nominalization, as proposed by Radford (2002). In mathematics, nominalization consists of a linguistic process where a given element is transformed into an algebraic expression, using an unknown quantity (the pattern number, in the case of arithmetic sequences) from the beginning of the process. The student must then perform operations based on this unknown quantity to arrive at an algebraic formula representing the situation. The second concept, introduced by Duval (2002), concerns the need to choose a letter to represent not just one, but several quantities expressed in a mathematical statement involving equations. For example, in the context of our suite, it may be necessary to designate the total number of tables and the number of middle tables by the same symbol, such as "n". Using this notation, the total number of tables would be represented by "n", and the number of tables in the middle would be represented by "n-2".

Symbolic algebraic generalisation: The secondary school pupils were the only ones to represent this type of generalisation. Figure 11 shows the final formula proposed by student 9



Translation:
The number of chairs required for a given number of tables

Figure 11: An example of the reasoning of student 9 who provided a symbolic algebraic generalisation

This student developed an expression for an action system of the type 10n - 2(n-1). He then had the idea of simplifying this expression. However, although his result was correct, it was still too strongly influenced by the actions performed. Then, he seemed to detach himself from the initial context to consider this expression in a more general way, like any other algebraic expression. This detachment from the context meant that the relationship with the initial problem disappeared, giving way to a purely mathematical expression. According to Dörfler (1991), this process can be seen as the "detachment of invariants from their original vehicle". Radford (2008) sees this stage as symbolic algebraic generalisation, where the expression evolves into a valid and autonomous mathematical form, independent of the original situation in the sequel.

5. Discussion and Conclusion

The aim of this article was to examine how Moroccan pupils in the sixth year of primary school and the first year of secondary school used their reasoning to generalise the term of an arithmetic sequence. This study is based on Dörfler's (1991) model, extended by Squalli (2015). Our contribution consists in exploiting this model in primary and secondary school contexts to analyse the evolution of algebraic reasoning from the first years of schooling. It is notable that a large number of pupils, both in the sixth year of primary school and in the first year of secondary school, did not grasp the issues or the structure of the activity. They encountered difficulties as early as the third task in the first question in trying to identify the invariants needed to perform the required calculations.

Most of the pupils who succeeded in expressing formulae in mathematical language remained at the "contextual" or "factual" level. However, for pupils in the sixth year of primary school, this represents a significant step, given that this activity marks their first introduction to algebra. In this context, the contextual formula emerges from the action processes developed by the pupils and is aligned with the operations. It is important to note that in the first year of secondary school, only two students succeeded in achieving symbolic algebraic generalisation. It is possible that this percentage could be increased if this activity were supported to encourage the pupils' reflection or to bring out the concepts and symbolisations linked to algebraic thinking, either by intervening within the groups or by organising a pooling, depending on the pupils' academic level. These results are in line with those of Vlassis et al (2017), who also stressed the importance of pedagogical support in such activities. It seems that these students did not necessarily make a connection between the algebra usually taught in class and the proposed activity. However, it should be borne in mind that the generalisation activity in which the students were involved required a very different use of algebra from the one they were used to practising (Vlassis et all,2017). This approach consisted not only of applying an algebraic technique already taught in class, but above all of 'producing' a new algebraic expression, requiring a distinct understanding of algebraic language, particularly with regard to the meaning of equality, letters and, most importantly, the relationships between different symbols.

The results obtained show that this activity led to equivalent and meaningful numerical and algebraic expressions, thus offering students the opportunity to discuss algebraic equivalence, as advocated by Samson (2012). In addition, this activity encouraged students to formulate generalizations, even without resorting to the use of letters. This finding confirms the recommendations of researchers in Early Algebra, who suggest the development of algebraic thinking from primary school before the formal introduction of algebra. In this context, arithmetic activities with algebraic potential, in particular generalisation, are proposed to pupils at this level.

In summary, this research highlights the different facets of the algebraic generalisation process in the Moroccan educational context. The results underline the importance of providing pupils with arithmetic activities with algebraic potential from primary school onwards, while also highlighting the need for a balanced pedagogical approach that combines the use of concrete material with the development of a formal understanding of algebra. This exploration paves the way for future thinking on how to integrate algebraic generalisation more seamlessly into educational programmes, aligning the transition between arithmetic and algebra to facilitate progression and conceptual understanding in pupils.

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