An Integral Approach for Computation of Cost-Benefit and Returns to Investment in Education

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Abstract *The formulae for computation of cost benefit and return to investment in education given respectively as (2.1) for PV (present value of returns to investment in education) and (2.2) NPV (net present value of returns to investment in education) are reduced to integral form and* $(1 + r)^{-t}$ *transformed to exponential and logarithmic functions, and simple straight forward formulae proffered for the calculation of PV and NPV.*

Keywords: *Integral Approach; Cost-Benefit; Returns; Investment; Education*

Introduction

The computation of wages returns to investment in education is an important aspect of educational administration which is aimed at determining if the many years spent in school is worthwhile or profitable to the learner in comparable terms when viewed in relation to the alternative of involvement in working or wages earning engagement. Marglin (1963) proposed two alternative rate for estimation of private investments education namely: Social opportunity cost of public investment (displaces private investment) and social time preference of public investment – displaces social time preference.

The computation of return to investment was done with serious uncertainties and errors until a clear picture given by the work of Mincer (1974) avail scholars the opportunity for easy computation of the returns to education. Mincer (1974) created a simple function which made wages a function of the years of education followed by individuals thereby making it possible to estimate the interest one could be earned on investment in education. Romer (1990) pinpointed that these returns are essential for estimation of the possibility of implementing, and developing of new technologies for the purposes of long term economic growth. Barro and Sala-i-Martins (2004) considered human capitals based on these theories as laudable and vital goods which aid the implementation of new technology. This places the returns to education on the industrial parlance as a detector of the supply of skilled labour (Nelson and Phelps, 1966: Welch 1970; Acemoglu, 1998). The rating of returns to education as a very valuable instrument of determining the economic growth of a nation in terms of its importance in supply of labour – skilled labour also held by economic historians (Ravallon and Datt, 2002; Zanden, 2004).

Considering the importance of the returns to economics and social developments, it is therefore very important to provide clear and simple formula which can be used in estimating the returns and cost-benefits.

Existing Formulae

The following two formulae are popularly used in estimating returns to education (Aromolaran, 2002 and Mincer,1974).

$$PV = \sum_{t=s+1}^{T}(W_{st} - W_{ot})(1 + r)^{-t} - \sum_{t=1}^{S} W_{ot}(1 + r)^{-t} \quad . \; . \; . \quad (2.1)$$

$$NPV = \sum_{t=s+1}^{T}(W_{st} - W_{ot})(1 + r)^{-t} - \sum_{t=1}^{S}(W_{ot} + c)(1 + r)^{-t} \quad . \; . \; . \quad (2.2)$$

In the formulae W_{st} represents present wages, W_{ot} wages befor4e schooling, cost of schooling, C, rate of returns or discounting rate r, number of years in school S, T total number of working years after schooling and t is the age of the person (individual). PV represents the individual's present capital value while NPV stands for the individuals Net presents capital value.

With formulae the estimation is supposed to be done one after the other, year by year and summed up for a particular person. The summation will then cover the number of years the person has put in, in the labour market. This may be time consuming and labourious.

Modification of the Formulae

The modification of the formulae is achieve by transformation of $(1 + r)^{-t}$ to exponential form and then logarithmic form. Consider the expressions below:

Let $(1+r)^{-t} = e^{-nt}$

then $\log_e(1+r)^{-t} = \log_e e^{-nt}$

hence $-t \log_e(1+r) = -nt \log_e e$

$-t \log_e(1+r) = -nt$

and $\log_e(1+r) = n$

$n = \ln(1+r)$

$(1+r)^{-t} = e^{-t\ln(1+r)}$

As a result PV can now be presented as

$$PV = \sum_{t=s+1}^{T}(W_{st} - W_{ot})e^{-t\ln(1+r)} - \sum_{t=1}^{S} W_{ot}e^{-t\ln(1+r)} \quad . \; . \; . \quad (2.3)$$

while

$$NPV = \sum_{t=s+1}^{T}(W_{st} - W_{ot})e^{-t\ln(1+r)} - \sum_{t=1}^{S}(W_{ot} + c)e^{-t\ln(1+r)} \quad . \; . \; . \quad (2.4)$$

Considering integration as sum, then

$$PV = (W_{st} - W_{ot})\int_{s+1}^{T} e^{-t\ln(1+r)}dt - W_{ot}\int_{1}^{S} e^{-t\ln(1+r)}dt \quad . \; . \; . \quad (2.5)$$

$$NPV = (W_{st} - W_{ot})\int_{s+1}^{T} e^{-t\ln(1+r)}dt - (W_{ot} + c)\int_{1}^{S} e^{-t\ln(1+r)}dt \quad . \; . \; . \quad (2.6)$$

Equation (2.5) gives PV in integral form while (2.6) presents NPV, gives method of integration for the calculation of NPV.

The formulae can further be represented as

$$PV = p\int_{s+1}^{T} e^{-t\ln(1+r)}dt - q\int_{1}^{S} e^{-t\ln(1+r)}\, dt \quad . \quad . \quad . \qquad (2.7)$$

$$NPV = p\int_{s+1}^{T} e^{-t\ln(1+r)}dt - k\int_{1}^{S} e^{-t\ln(1+r)}dt \quad . \quad . \quad . \qquad (2.8)$$

where $p = W_{st} - W_{ot}$

$q = W_{ot}$

$k = W_{ot} + C = q + C$

since $n = \ln(1+r)$

then

$$PV = p\int_{s+1}^{T} e^{-nt}dt - q\int_{1}^{S} e^{-nt}\, dt \quad . \quad . \quad . \qquad (2.9)$$

$$NPV = p\int_{s+1}^{T} e^{-nt}dt - k\int_{1}^{S} e^{-nt}dt \quad . \quad . \quad . \qquad (2.10)$$

Hence, equations (2.7) and (2.9) give the formulae for computing the individual's present capital value of rewards or cost- benefit to education while (2.8) and (2.10) give the formulae for calculating the net present capital value of the rewards or cost-benefit to education using method of integration.

Analytical Outcome

The integral

$$\int_{S+1}^{T} e^{-nt}\, dt = \frac{e^{nT} - e^{n(S+1)}}{ne^{n(T+S+1)}} \quad . \quad . \quad . \qquad (2.11)$$

while the integral

$$\int_{1}^{S} e^{-nt}\, dt = \frac{e^{n(S-1)} - 1}{ne^{nS}} \quad . \quad . \quad . \qquad (2.12)$$

Utilizing the equations (2.11) and (2.12) , the PV and NPV are given as:

$$PV = p\left(\frac{e^{nT} - e^{n(S+1)}}{ne^{n(T+S+1)}}\right) - q\left(\frac{e^{n(S-1)} - 1}{ne^{nS}}\right) \quad . \quad . \quad . \qquad (2.13)$$

while

$$NPV = p\left(\frac{e^{nT} - e^{n(S+1)}}{ne^{n(T+S+1)}}\right) - k\left(\frac{e^{n(S-1)} - 1}{ne^{nS}}\right) \quad . \quad . \quad . \qquad (2.14)$$

The integral

$$\int_{S+1}^{T} e^{-nt} dt = \frac{e^{nT} - e^{n(S+1)}}{ne^{n(T+S+1)}} \quad . \quad . \quad . \quad (2.11)$$

while the integral

$$\int_{1}^{S} e^{-nt} dt = \frac{e^{n(S-1)} - 1}{ne^{nS}} \quad . \quad . \quad . \quad (2.12)$$

Utilizing the equations (2.11) and (2.12) , the PV and NPV are given as:

$$PV = p\left(\frac{e^{nT} - e^{n(S+1)}}{ne^{n(T+S+1)}}\right) + q\left(\frac{e^{n(S-1)} - 1}{ne^{nS}}\right) \quad . \quad . \quad . \quad (2.13)$$

while

$$NPV = p\left(\frac{e^{nT} - e^{n(S+1)}}{ne^{n(T+S+1)}}\right) - k\left(\frac{e^{n(S-1)} - 1}{ne^{nS}}\right) \quad . \quad . \quad . \quad (2.14)$$

Thus 2.13 and 2.14 offer direct formulae extracted from method of integration for computation of PV and NPV with ease. As reminiscence, the constants in the formulae are defined as follows:

$p = W_{st} - W_{ot}$
$q = W_{ot}$
$k = W_{ot} + C$
$n = 1 + r$

s,T,t,r, Wot, Wst and C are as been defined earlier in which case

S	=	Duration of schooling (years in school)
T	=	Number of working years (years on employment)
t	=	Age of the individual
r	=	Discounting rate or rate of returns
C	=	Cost of schooling
W_{ot}	=	Salary (wages) before schooling
W_{st}	=	Salary (wages) after schooling

The figures can be collected from interview and with the help of questionnaire or individuals' work file.

Bases for Modification to Integral and Alternative Formulae

The fundamental reasons for the modification of the formulae for computing the PV in equation (2.1) and the NPV in equation (2.2) are basically that integral

$$\int_a^b y\,dx = \int_a^b f(x)\,dx$$

can be expressed as a sum and that

$$(1+r)^{-t} = e^{-nt}$$

where $n = \log_e(1 + r)$

and $(1+r)^{-t} = e^{-t\ln(1+r)}$

is continuous, as such integrable. Consider the definition of the values given thus:
let Y be a function θ (x) of x and suppose that the range from x = a to x = b is divided into n equal sub-ranges each of width δx. Let $Y_1, Y_2, Y_3, \ldots, Y_n$ be the values of Y at the middle points of each sub-range. The arithmetic mean of those n values of Y is

$$\frac{(Y_1 + Y_2 + Y_3 + \ldots + Y_n)}{n}$$

Since nδx = b – a, this can be

$$\frac{(Y_1 + Y_2 + Y_3 + \ldots + Y_n)\delta x}{b - a}$$

If as n → x or δx → 0 the expression has a limiting value, the limit is

$$\frac{\int_a^b y \; dx}{b - a}$$

and this called mean value (see Tranter, 1978). Going by the above

$$\sum_{t=s+1}^{T} (W_{st} - W_{ot})(1 + r)^{-t} \quad \text{and}$$

$$\sum_{t=1}^{S} (W_{ot} + c)(1 + r)^{-t}$$

are justifiable integrable with the given limits. For obvious reason $(1+r)^{-r}$ is a continuous function and can be integrated, since r = discounting rate or rate of returns is always > 0, hence the adoption of the method of integration.

Analytical Comparison

Considering wages before schooling W_{ot} of N200000.00 per annum; wages after schooling W_{st} of N12000000.00 per annum; duration of schooling of four (4) years; rate of returns, r of 0.2 or 20%; and cost of schooling of N50,000.00 the PV and NPV are computed as

(a) $$PV = \sum_{t=s+1}^{T}(W_{st} - W_{ot})(1 + r)^{-t} - \sum_{t=1}^{S} W_{ot}(1 + r)^{-t}$$

(b) $$PV = p\left(\frac{e^{nT} - e^{n(S+1)}}{ne^{n(T+S+1)}}\right) - q\left(\frac{e^{n(S-1)} - 1}{ne^{nS}}\right)$$

(c) $$NPV = \sum_{t=s+1}^{T}(W_{st} - W_{ot})(1 + r)^{-t} - \sum_{t=1}^{S}(W_{ot} + c)(1 + r)^{-t}$$

(d) $$NPV = p\left(\frac{e^{nT} - e^{n(S+1)}}{ne^{n(T+S+1)}}\right) - k\left(\frac{e^{n(S-1)} - 1}{ne^{nS}}\right)$$

Gives PV of N220000.00 and NPV of N90000.00 approximately for T = 7. Hence the methods are reliable, only that formulae (a) and (c) are tedious and time consuming especially when larger values of T are involve. This make (b) and (d) preferable.

Conclusion

Considering the importance of cost-benefit and return to education in national and individual planning as well as educational administration, policies and decision making, it is pertinent to avail to quiescence formulae and methods for computation that can be clearly understood and easily used by all stakeholders. In this light, the method of integration and the alternative formulae given above are offered by this paper. They will be of immense help to those who care to calculate the cost benefit and returns to investments in education.

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