

## Developing a Theoretical Model for Investigating the Mathematics and Science Teachers' PCK in South Africa and Zimbabwe

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Doi:10.5901/mjss.2013.v4n14p473

### Abstract

*This paper presents a theoretical framework comprising components of teacher knowledge that can render effective learner understanding of mathematical and science concepts. Twelve mathematics teachers at the secondary school level were selected from both countries on the basis of their school consistent good performance for at least three years. The study adopted a qualitative research method. The data on the teachers' PCK were collected through lesson observations, questionnaires, interviews, video recordings, teachers' written reports, and document analyses. An iterative coding and categorisation were used to identify the emerging theme of the study. The result of the study shows that the emerging theme that form the framework can inform mathematics and science instruction to balance the ternary relationship between curriculum content, learners and teacher quality in terms of pedagogical practice. The framework was found promising in either study but a more conclusive evidence will be given after a planned longitudinal study.*

### 1. Introduction

The results of the Third International Mathematics and Science Study (TIMSS) have indicated poor learner performance in many countries around the world (Reddy, 2006). Although Zimbabwe was not among the 1998 and 2003 participating countries in TIMSS, neighbouring South Africa and Botswana were ranked among the bottom five performing countries. This disappointing performance by neighbouring South Africa and Botswana was perceived as proxy of Zimbabwe's performance. The performance was perceived by educators and policy makers in the region as reflecting performance of learners in Southern Africa. A Sub-Saharan regional systemic assessment, Southern and Eastern Africa Consortium for Monitoring Educational Quality (SEACMEQ), regularly assess learners' performance in mathematics and science as away of monitoring the quality of teaching in these areas. Of the fifteen countries participating in the SEACMEQ systemic assessment the performance of learners from the regional powerhouse, South Africa, was ranked 10th behind relatively poor countries such as Tanzania, Kenya, Swaziland and Zimbabwe. The SEACMEQ and TIMSS assessment results corroborated to show that poverty is not a factor in educational performance. This observation provided fertile imaginations, speculations and hypothesizing that factors that may influence educational quality are likely to be deeply rooted in the learner, national curriculum, subject matter and pedagogical flexibility of teachers. The present paper reports on a theoretical framework that was developed to guide teachers' understanding of the ternary relationship between learners, curriculum content and pedagogy.

### 2. Contexts of South African and Zimbabwean Mathematics and Science Curricula

The current South African and Zimbabwean school curriculum are based on similar philosophical bases of the objectives model. The Zimbabwean school curriculum is visibly based on Ralph Tyler's four considerations of purposes, experiences, organization and evaluation of the experiences (Bilbao 2008). The South African national curriculum is based on outcomes based education (OBE). In its broad sense OBE is a process involving the restructuring of curriculum, assessment and reporting practices in education to reflect the achievement of high order learning and

mastery of knowledge and skills rather than the accumulation of isolated content (Tucker 2004).

### 3. Teacher Knowledge

Teaching mathematics and science is a complex enterprise (Ellison, 2007, Schoenfield, 2005) involving both cognitive and socio- systemic factors (Jaworski, 2004). Effective teaching of these subjects requires wise choice of resources, suitable pacing and sequencing of content, creating classroom environments that promote learner participation, selecting activities that motivate, challenge and extend learner thinking and understanding the ever changing learner interests, attitudes and dispositions (Even, 2005). The teaching and learning of mathematics and science are complex enterprises about which much remains to be revealed and understood. Content and pedagogical knowledge are different aspects which are used separately in planning and teaching. Teachers need not possess "just content knowledge that many educated adults have, but also knowledge specialized for teaching (Izsak, 2006)". It is in the light of the importance of these components of the teaching and learning of mathematics and science that analyses of blending pedagogical and content knowledge are necessary to produce learner understanding. Shulman (1987) referred to this knowledge as pedagogical content knowledge (PCK). PCK is defined as "the blending of content and pedagogy into an understanding of how particular topics, problems or issues are organized, represented, and adapted to the diverse interests and abilities of learners (Shulman, 1987:12)". PCK identifies the distinctive bodies of knowledge for teaching that distinguish the understanding of the content specialist from that of teachers. Key in distinguishing the knowledge base for teachers and that of subject specialists lies in "the intersection of content and pedagogy", in one's ability to transform the content knowledge possessed into "forms that are pedagogically powerful and yet adaptive to the variations in ability and background of a learning context and peculiar school factors (Shulman, 1987)".

What is apparently clear in the analysis of PCK is a teacher's possession of special attributes that can help him/her to guide learners to understand and master content in a way that is personally meaningful. A teacher's effective PCK is inferred in learners' ability to communicate, reason, apply and transfer classroom content in various facets of their environments and other disciplines in the school curriculum. This inference makes sense because effective teaching is largely defined in relation to learning where student knowledge is gained in conducive classroom environments (Ellison, 2007). This implies that effective teaching is determined in part by a teacher's knowledge of learners' interests, prior knowledge and cognitive development which a teacher uses to select cognitively challenging content, activities and pacing of a lesson among other factors. A lot of research on PCK focused on teaching and student learning (Ellison, 2007) but with little focused on what teachers with deep content and pedagogical knowledge about learners can do and how they use what they know to inform teaching and learning practices. For instance, when does a constructivist inspired teacher use inquiry pedagogical and when does he/she use traditional methods during mathematics or science instruction? Answers to this question are influenced by teachers' instructional conceptions and knowledge of the learning context. Conceptions are personalized instructional ideas and imaginations that teachers hold about the nature of mathematical/scientific knowledge and how it can be mastered by students. A teacher's conceptions mould the assumptions, methodology and presentation of mathematical/scientific knowledge in ways that are believed to make students understand the subject content (Nyaumwe, 2004). The present study sought understanding of how mathematical and scientific teachers can organize, present, adapt and transform particular topics and concepts into representations that make sense to learners of diverse interests and abilities.

The study extended (Shulman, 1987) PCK model to include learner factors. This approach is consistent with other researchers who extended the PCK model, for instance to include educational technology (Koehler and Mishra 2009). The theoretical model designed for the longitudinal study is shown in Figure 1 below.

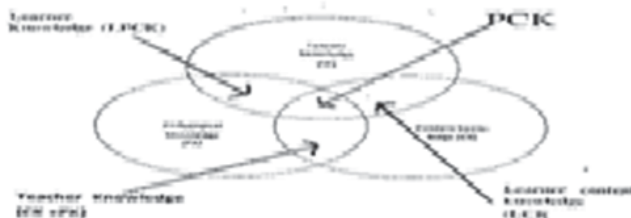


Figure 1: Components of teacher knowledge that can render effective learner understanding

Each of the cells in the figure is explained using the context of the study.

#### **4. Learner knowledge (LK)**

Learners come in the mathematics and science classroom with a host of attitudes, dispositions, interests and expectations that varies from one day to the next and can fluctuate within the same lesson. Teaching them effectively entails understanding their affective needs and choosing examples that can motivate them to learn. Failure to understand their affective and psychological needs may breed resistance to learn that is usually shown through disruptive behaviour, lack of concentration and not paying attention to what is being covered in a lesson. A teacher's learner knowledge influences the choice of examples and structuring learning concepts to a level that meets learner needs.

#### **5. Learner content knowledge (LCK)**

Mathematics and science subjects are hierarchically cumulative disciplines in which learners' prior knowledge is central to the building of new concepts. Effective teaching of these subjects entails diagnosing learners' misconceptions and their origins, how learners construct knowledge, acquire new skills and how they develop habits of mind that are aligned to mathematical and scientific thinking and positive dispositions. Achievement of these aspects can lead to connections of concepts in the same subject or across disciplines, selecting multiple representations of concepts, choosing alternative teaching strategies aligned to learners' different learning styles, and exploring alternative ways of looking at the same concept.

#### **6. Content knowledge (CK)**

Effective mathematics and science teachers need strong content knowledge in order to be flexible in presenting it and to cognitively challenge learners' thinking. Teacher deep content knowledge enables them to change their teaching plans based on learners' competencies (or lack of them) so that learning activities are aligned to learners' prior knowledge through scaffolding and designing extension work during teaching. The content knowledge that teachers possess is different from that possessed by subject specialists, not necessarily in the quantity and quality, but in how that knowledge is organized and used (Ijeh, 2013). Where as scientists' and mathematicians' content knowledge is structured from a research perspective to facilitate construction of new knowledge, teachers' knowledge is structured on a teaching perspective to enhance learners' understanding (Mark, 1990; Ijeh, 2013). This means that teachers' knowledge should be flexible enough to be expressed in multiple representations that suit the pedagogical strategies that are possible or teachers to use in a lesson.

#### **7. Pedagogical content knowledge (PCK)**

Pedagogical content knowledge is the intersection of content and pedagogy. The intersection influences a teacher's "capacity to transform the content knowledge possessed into forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by students (Shulman, 1987)". Developing a synergy of mathematical and scientific instructional repertoire entails tracing variations of learners' conceptual understanding and detecting possible sources of learners' misconceptions (if any) which underscores the fact that the art of teaching is best mastered by real teaching. Theoretical knowledge that is not embedded in teaching practice that gives teachers opportunities to build personalized teaching knowledge and skills using classroom contexts is not enough for effective teaching (Nyaumwe, 2004).

#### **8. Pedagogical knowledge (PK)**

In the last century (Thom, 1973) posited that "all pedagogy, even if scarcely coherent, rests on a conception of knowledge acquisition." This means that teachers' pedagogical choices are informed by the conceptions that they hold about the nature of scientific and mathematical knowledge. Teachers holding the traditional conception of mathematical and scientific knowledge believe that concepts in the subjects are fixed, universal and unchanging and mastering legitimate process skills entails regurgitating procedures using established axioms, definitions and theorems (Lubisi, 1997). On the other hand teachers holding constructivist conceptions of teaching believe that scientific and mathematical knowledge

are tentative, intuitive, subjective, and dynamic and that they originate from observation, experimentation and abstraction using senses (Davis, 1990). Teachers need deep understanding of a variety of pedagogical knowledge from the traditional and constructivist paradigm in order for them to interpret learning contexts created by learners and adapt teaching methods that maximize learner understanding.

## 9. Learner pedagogical knowledge

Learner pedagogical knowledge is the ability of teachers to understand their students and the pedagogical approaches that may suit them. Teachers' learner pedagogical knowledge is important to enable them to interpret students' learning styles and aligning them to pedagogical approaches that suit individual students in a class.

## 10. Learner Pedagogical content Knowledge (LPCK)

Learner pedagogical content knowledge comprise of teachers' understanding of learners, pedagogy and content and their efforts to integrate the three aspects during teaching. This entails the sensitivity that is necessary to reflect during teaching and post teaching on the suitability of the pedagogical approaches used to teach the content at hand and a critical assessment of learner understanding. Teaching science and mathematics is a personal craft that requires teachers to reflect on their pedagogical practices in order to assess the effectiveness of the pedagogical strategies they implement in different classroom environments.

## 11. Methodology

This study adopted a review of the related literature on pedagogical content knowledge for teaching mathematics and sciences at secondary school level. The review focused on teacher knowledge, learner knowledge, content knowledge, learner content knowledge, pedagogical content knowledge, pedagogical knowledge, learner pedagogical knowledge and learner pedagogical content knowledge. Based on analysis and review of related literatures on PCK, a theoretical model was derived through coding and categorization of data obtained from the reviewed literature on PCK. The themes that emerge from the reviewed literature were later evaluated using the instruments and data collected on teachers' PCK in statistical graphs and projectile motion adopted from the study of Ijeh (2013) and Awelani (2013) to justify its validity and reliability of the choice of the theme as a theoretical framework for studying teachers' PCK in mathematics and sciences. The instruments that were used to collect data to attest the PCK model were teacher conceptual knowledge and concept map exercise, questionnaire, interview, lesson observation and documents analysis.

## 12. Result and discussion

### 12.1 Subject matter content knowledge in mathematics and science

To evaluate the theoretical framework of this study is to determine to what extent the theoretical framework has enabled the researchers involved in this study to determine teachers' PCK in mathematics and science. The process of evaluating the theoretical framework started with the determination of mathematics and science content knowledge of participating teachers using various instruments adopted from the study of Ijeh (2013) and Awelani (2013) on how teachers develop their PCK in statistics and projectile teaching.

The conceptual knowledge exercise, concept mapping exercise, teacher interviews, document analysis were the instruments used to examine the subject matter content knowledge of the participating teachers in school statistics and mechanics in this study. The intention of the researcher in using these instruments for data collection was to determine the subject matter content knowledge that the participating teachers demonstrated in classroom practice. What can be gleaned from the results is that the instruments allowed the researcher to capture the teachers' PCK in terms of the subject matter content knowledge in statistics and projectile motion teaching. The concept map exercise was used as a proxy, but was not sufficient to determine how knowledgeable the teachers were about the contents of the mathematics and science curriculum. The teachers should have been requested to write an examination in order to determine their content knowledge of the topic. However, a concept mapping exercise was considered a good proxy for assessing their content knowledge. Another way in which the teachers' content knowledge could have been examined was through certification. That is by reviewing the certificate obtained from colleges and universities. Considering a certificate in

mathematics education without observing how a teacher demonstrates his or her content knowledge in the classroom may not be sufficient to determine whether that teacher possesses content knowledge of a topic. Hence, lesson observations were used to assess the teachers' subject matter content knowledge and how well they demonstrated this knowledge in mathematics and science teaching. Although Mahvunga and Rollnick (2011) suggest that a quantitative research study may be sufficient to assess teachers' content knowledge, their study failed to indicate how to assess the quality of teachers' content knowledge, which can be determined only during classroom practice. This assertion is given wide empirical support by researchers such as Toerien (2011), Ball *et al* (2008), Capraro *et al* (2005), Jong *et al* (2005), Lee and Luft (2008), Jong (2003) and Gess-Newsome and Lederman (2001), who all note that PCK is rooted in classroom practice. Any research into teachers' PCK that does not consider the use of lesson observation may fail to fully convey the required information about how teachers develop topic-specific PCK. Through lesson observation, it was possible to determine how the teachers demonstrated their content knowledge of the curriculum and pedagogy of certain topics. Lesson observation provided opportunities to experience the details, nuances and dimensions that the teachers used in their classroom practice in order to determine the adequacy of their subject matter content knowledge of the curriculum and pedagogy. Through the teacher interviews, it appears that the teachers' educational backgrounds that may have enabled them to develop topic-specific content knowledge in mathematics and science were determined.

While researchers such as Shulman (1986), Van Driel *et al* (1998), and Magnusson *et al* (1999) use subject matter knowledge consisting of syntactic and substantive knowledge acquired in formal education, in this study the subject matter content knowledge of the curriculum focused on the content to be taught and learned by the students. The use of curriculum content as the theoretical framework for this study proved useful in determining the procedural and conceptual knowledge (component of the PCK) that a teacher demonstrates in teaching mathematics and science. Other PCK studies (Jong, 2003; Ibeawuchi, 2010; Ogbonnaya, 2011; and Toerien, 2011) share the same view of using subject matter content knowledge as a theoretical framework for examining teachers' PCK development in mathematics. These authors also assess the subject matter by making the teachers write a test on the content of the topic under investigation. The instruments developed with the framework were therefore considered adequate to determine teachers' subject matter content knowledge in statistics and mechanics teaching and the theoretical framework can be considered adequate and valid.

### 12.2 Pedagogical knowledge (Instructional skills and strategies)

The teacher questionnaire, which focused on what the teachers did while teaching the assigned topic, and the written reports used to triangulating the data collected with lesson observations were used to determine the pedagogical knowledge (instructional skills and strategies) that the teachers used in teaching school statistics. Other instruments used to assess the teachers' pedagogical knowledge were lesson observation and document analyses.

The questionnaire revealed many aspects of the teachers' PCK, such as knowledge of instructional skills and strategies for teaching statistical graphs. These strategies included oral probing questioning, checking and marking learners' homework and pre-activities to determine learners' pre-existing knowledge. The lesson observations, teacher written reports and document analyses confirmed the use of these instructional strategies. These activities were crucial in determining learners' conceptions about statistical graphs, as suggested by Krebber (2004), Westwood (2004) and Ball *et al* (2008), but did not elicit learners' preconceptions in statistical graphs and projectile motion. From the lesson observations, it was not possible to determine learners' preconceptions because the strategies the teachers adopted to do so did not elicit them. Instead, the learners displayed previous knowledge linked to learning the new topic. In fact, the teachers did not have knowledge of the instructional skills and strategies that might have been necessary to determine the learners' preconceptions in statistical graphs and projectile motion.

As Krebber (2004) and Westwood (2004) suggest, the use of the instructional strategies of oral probing questioning, pre-activities, checking and marking learners' responses to classwork, homework and examining learners' understanding, as well as identifying their misconceptions and learning difficulties in statistical graphs and projectile motion, is critical in learning and could motivate the development of teachers' pedagogical knowledge. Loughran *et al* (2004), Ball *et al* (2008), and Vistro-Yu (2003) regard teachers' pedagogical knowledge as crucial to PCK development. Having ascertained the instructional skills and strategies demonstrated by the teachers through the teacher questionnaire, written reports, document analyses and lesson observation, the researcher believes that the teachers' pedagogical knowledge can be considered a valid theoretical framework for determining the PCK required for teaching school statistics.

### 12.3 Learners' Misconceptions and learning difficulties

However, the framework provided an opportunity to reveal that the teachers had some knowledge of learners' misconceptions, as individually they were able to identify misconceptions through analysis of learners' responses to classwork, homework and assignments in statistical graphs. The activities of identifying and addressing learners' misconceptions are critical aspects of teaching and learning. Penso (2002), Ibeawuchi, 2010 and Westwood (2004) note that a teacher who lacks the ability to identify and address learners' misconceptions may experience poor content delivery in classroom practice. Practising mathematics teachers are encouraged to learn about the possible instructional skills and strategies for identifying and addressing learners' alternative conceptions in statistical graphs and projectile motion.

### 13. Conclusion

The sizes of the four intersections that are created by the three circles symbolising learners, content and pedagogy are influenced by the experience of a teacher. Experienced teachers show greater overlaps which symbolise increased integration of the three whilst smaller overlaps depict minimal integration typical of novice teachers' practice (Mark, 1990). This means as teacher educators we need to make effort to assist preservice and INSET teachers so that this overlap can be increased. The effectiveness of the framework will be explored in a longitudinal study to follow.

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