

Testing Applicability of Value at Risk Models in Stocks Markets¹

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Abstract

This paper evaluates the forecasting performance of Value at Risk (VaR) method based on two wide spread approaches, historical simulation and Risk Metrics, before and after the sub-prime crisis in the context of developed and emerging capital markets. We present results on both VaR 1% and VaR 5% on a one-day horizon for Belex 15 and SAX. For comparative purposes, the paper also focuses on the DJIA and the STOXX Eastern Europe Total Market Index, an index representative of emerging European stock markets. In order to validate accuracy of VaR results we employ different back test techniques. Results indicate that the relative performance of VaR as a measure of market risk significantly underestimates the true level of market risk in Serbian stock market, in contrast to Slovak, where standard VaR approaches accurately capture market risk exposure. Results also provide evidence that the characteristic of stock markets and their asset returns in combination with the desired confidence level and risk horizon determine how well a certain approach performs on a certain stock market.

Keywords: VaR, emerging stock market, backtesting, market risk

1. Introduction

The term emerging markets is commonly used to describe an economy with a GDP per capita substantially below the advanced world average and typically with a growth potential above the global average. According to the World Bank's definition an emerging markets country has a Gross National Income per capita less than approximately USD 9,000. Within this category differences between individual countries vary a lot in terms of economic governance and institutional framework, infrastructure, physical size and political regime. There is no key definition to describe an emerging markets economy. Over time a country can mature to be categorized an advanced economy while others can emerge from unrated illiquid poorly regulated frontier markets status to become emerging markets economies included in global indices. Since the mid-1990s emerging and developing countries have had growth rates above the global average, especially over the last decade. The most recent crisis (2008/2009) has helped to underline the emergence of emerging economies as the economic powerhouse of the world.

Emerging countries are characterized by a market economy that is in-between developing and developed status. These countries frequently experience very high volatility and extreme movements in their stock markets. Examples of such highly volatile periods include the Mexican debt crisis in 1994, Asian crisis in 1997, Russian default in 1998, the Turkish banking crisis in 2001, global financial crisis in 2008 and current EUROZONE debt crisis. (Ergen, 2010)

According to the aforementioned examples of markets volatility, most of European countries experienced financial distress, and these emerging markets suffered million losses due to poor financial risk management. Financial risk includes situations where there is uncertainty and opportunity for achievement's financial loss or in other words, this risk

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is related to the money that can be lost in the financial markets. The development of the financial sector makes a lot of innovation in the field of financial risk management, especially in the modeling of market, credit and operational risk.

The aim of this paper is to capture and evaluate relative performance of market risk measures in two emerging stock markets: Slovakia and Serbia, in the period of 2005 to 2012. Reason why we chose those markets is threefold.

Firstly, Serbia and Slovakia are similar by majority economic criteria and it is interesting to compare accuracy of same VaR methodologies in Belgrade and Slovak stock exchange, considering that Slovakia is an EU member state and Serbia is an EU candidate. Secondly, both financial markets are banking orientated economies in terms of asset size and their share in total assets of financial institutions exceeds eighty percent (in Slovakia even over ninety percent) (Živković, 2007). Thirdly, the idea is to extend the very scarce empirical research on VaR estimation in those financial markets.

For comparative purposes, the same risk models are applied to data obtained for the US capital market, as represent of efficient market, and STOXX Eastern Europe Total Market Index (TMI), as represent of international portfolio of European emerging stock markets. Reason for choosing this index is twofold. First, we examine the benefit of international diversification on VaR measure. Second, two countries, Slovakia, Serbia, are part of the aforementioned index. The STOXX TMI index represents the Eastern European region as a whole. With a variable number of components, it covers approximately 95 percent of the free float market capitalization of 18 Eastern European countries. The STOXX Eastern Europe TMI Index comprises large, mid and small capitalization indices.

The main measure of market risk that would be applied in this paper is Value at Risk (VaR) methodology. Therefore, a precise estimate of VaR is a critical point in risk management. The aim of VaR is to provide a single number summarizing the total risk in a portfolio of assets due to changes in market quotes. However, whether we are talking about research of domestic or foreign authors, analysis of VaR performance are mostly based on the developed capital markets. There are rare cases of risk assessment of movements in stock prices using VaR methods in the Serbian and Slovakia capital markets, according to our last knowledge.

The purpose of this paper is to test VaR models in observed equity markets before and during the global financial crisis. It is interesting how models for market risk assessment, created and most widely used in developed equity markets, estimate financial risks in emerging markets. Contribution of this paper is that no one so far has investigated relative performance of VaR and compared the financial data obtained by this model in the Serbian and Slovakia stock markets during the observed risk horizon. Another contribution is to extend the very scarce empirical research on VaR estimation in emerging financial markets.

The ongoing global financial crisis caused problems in risk factor distribution even in US stock market, which is a benchmark for an efficient capital market. In order to use models to assess market risk, it is very important to evaluate potential shape and behavior of indices returns distribution, since VaR models are known to be accurate only under normal market conditions. Normality and independence are important properties from a risk modeling perspective, which will be examined in paper.

The financial crisis and its aftermath clearly resulted in variations in predicted risk. Our final goal is to show that using of two well known VaR models, Historical simulation (HS) and Risk Metrics (RM) approaches, can seriously underestimate market risk in both developing and especially in emerging stock markets during market stress circumstances.

These two models can be used to estimate market risk but they must satisfy two properties: unconditional coverage and independence, for model validation purposes. The Basel Committee (1996) has set up a regulatory back testing framework in order to monitor the frequency of exceptions but, due to the simplicity of the test, there is hardly a reason to use it in model validation processes when there are more powerful approaches available.

The rest of the paper is organized as follows: Section 2 presents a brief literature review of VaR and recent similar researches. Section 3 present descriptions of tested VaR models with emphasis on HS and RM models and back testing techniques. Section 4 gives the description of the analyzed data and statistical characteristics of all analyzed stock markets. Findings and back testing results are also presented and discussed in section 4. Section 5 concludes.

2. Literature Review

The first clues about the origin of the theory of portfolio were given by Hardy and Hicks. They discussed intuitively the merits of diversification (Hardy, 1923), (Hicks, 1935). Leavens gave a quantitative example, which was the first VaR measure ever published (Leavens, 1945). Leavens considered a portfolio of ten bonds over some horizon. Leavens did not explicitly identify a VaR metric, but he mentioned repeatedly the "spread between probable losses and gains." He seems to have had the standard deviation of portfolio market value in mind. Markowitz and Roy independently published VaR measures to optimize reward for a given level of risk (Markowitz H. M., 1952), (Roy, 1952). Because of the limited

availability of processing power, VaR measures from this period were largely theoretical, and were published primarily in the context of the emerging portfolio theory. This encompassed the work of (Tobin, 1958), (Treyner, 1961), (Sharp, 1964), (Lintner, 1965) and (Mossin, 1966). The VaR measures they employed were best suited for equity portfolios.

As the 1970s turned to the 1980s, markets were becoming more volatile. Firms were becoming more leveraged, and the need for financial risk measures, such as VaR, was growing. The resources to implement VaR were becoming available, but VaR remained primarily a theoretical tool of portfolio theory. Firms needed some way to measure market risk across disparate asset categories, but did not recognize how VaR might fill this need. US regulators were laying the groundwork for them to do so.

By 1993, a fair number of financial firms were employing proprietary VaR measures to assess market risk, allocate capital or monitor market risk limits. The measures took various forms. The most common approach generally followed (Markowitz H. M., 1959). Wilson represents the first published attempt to reflect leptokurtosis and heteroskedasticity in the practical VaR measures used on trading floors (Wilson, 1993). It is also the first detailed description of a VaR measure for use in a trading environment since (Garbade, 1987) paper.

Several similar names were used during the 1990's, including: "dollars-at-risk" (DaR), "capital-at-risk" (CAR), "income-at-risk" (IaR), "earnings-at-risk" (EaR) and "value-at-risk" (VaR). Finally, Guldumann suggests that the name "value-at-risk" (Guldumann, 2000). There were criticisms suggesting that VaR measures were conceptually flawed (Taleb, 1997).

An overview of Value at risk especially in scenario analyses which treats the potential losses associated with particular scenarios, such as a parallel shift of a given change in volatility were given by (Duffie & Pan, 1997). Marshall and Sigel found that failures of risk management have led to a widespread call improved quantification of financial risk (Marshall & Sigel, 1997). The problem of applying VaR in practice was evident because empirical returns distributions tend to have tails that look quite different from those of the normal and lognormal distributions that we typically assume in finance (Nefchi, 2000).

Measuring of market risk at the Serbian Belgrade Stock Exchange (BSE) has not been as extensively studied as some countries of former Republic Yugoslavia. Živković tests a wide range of VaR models on transitional markets of 2004 and 2007 EU new member states as well as EU candidate states. Findings show that VaR models commonly used in developed stock markets are not well suited for measuring market risk in these markets (Živković, 2007). His contribution is significant in this scientific field, but testing is done before the global financial crisis. Kovačić successfully applied volatility forecasting models on the Macedonian stock market index in an effort to measure relationship between return and conditional volatility (Kovačić, 2007). Terzić et al discovered the main commonly known stylized facts in the Serbian stock market data and quality of different market risk measures (Terzić & Milojević, 2013). Radivojević et al (2009) tested the applicability of parametric and non-parametric VaR methods in Serbian stock market and founded that parametric VaR is reliable to assess market risk in BSE, but we think their findings are not correct, because they used short risk horizon and only one simplest backtest to validate VaR model (Radivojević, Milojković, & Stojković, 2010).

The advantage of VaR method in institutions from micro to macro level has studied by Klištić (Klištić, 2011). Extreme Value Theory and VaR methodology are good risk measurement tools for extreme events and especially for high volatility times (Avdulaj, 2010).

VaR methodology is widely applied in banking sector of EU countries due to Basel Rules (Barjaktarović, Filipović & Dimić, 2013). At the same time it did not provide competitiveness of Serbian banking sector as part of CEE region (Barjaktarović & Ječmenica, 2011).

3. Methodology of Research

Before describing VaR back tests in any detail it is important to have a concrete definition of VaR in mind. From a mathematical point of view, VaR is the maximum loss that we will suffer over a certain time period with a predetermined probability level $q = 1 - \alpha$, called the confidence level. Formally, VaR at level q is the real number x such that:

$$P(X \leq x) = \int_{-\infty}^x f(x) dx = 1 - \alpha, x = VaR(q) \quad (1)$$

where $f(x)$ is the density function of the loss distribution. Generally speaking, with deviation from normality assumption, there are two main methods to construct VaR: parametric approach (analytical approach) and the nonparametric approach (historical simulation approach) (Dowd, 2002).

The HS technique is deceptively simple. Consider the availability of a past sequence of m daily hypothetical portfolio returns, calculated using past prices of the underlying assets of the portfolio, but using today's portfolio weights; call it $\{R_{pf,t+1-\tau}\}_{\tau=1}^m$. The HS technique simply assumes that the distribution of tomorrow's portfolio returns, $R_{pf,t+1}$, is

well approximated by the empirical distribution of the past m observations, $\{R_{pf,t+1-\tau}\}_{\tau=1}^m$. Put differently, the distribution of $R_{pf,t+1}$ is captured by the histogram of $\{R_{pf,t+1-\tau}\}_{\tau=1}^m$. The VaR with coverage rate p , is then simply calculated as 100 p th percentile of the sequence of past portfolio returns (Christoffersen, 2012). We write:

$$VaR_{t+1}^p = -Percentile\left(\{R_{pf,t+1-\tau}\}_{\tau=1}^m, 100p\right) \quad (2)$$

Thus, we simply sort the returns in $\{R_{pf,t+1-\tau}\}_{\tau=1}^m$ in ascending order and choose the VaR_{t+1}^p to be the number such that only 100 p % of the observations are smaller than the VaR_{t+1}^p . As the VaR typically falls in between two observations, linear interpolation can be used to calculate the exact number. Standard quantitative software packages will have the Percentile or similar functions built in so that the linear interpolation is performed automatically (Alexander, 2008).

The RM approach is a particular, convenient case of the GARCH process. Variances are modeled using an exponentially weighted moving average (EWMA) forecast. The forecast is a weighted average of the previous forecast, with weight λ , and of the latest squared innovation, with weight $(1 - \lambda)$:

$$\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1 - \lambda)r_{t-1}^2 \quad (3)$$

The λ parameter, also called the decay factor, determines the relative weights placed on previous observations (Jorion, 2002). The EWMA model places geometrically declining weights on past observations, assigning greater importance to recent observations. The weights decrease at a geometric rate. The lower λ , the more quickly older observations are forgotten. Risk Metrics has chosen $\lambda = 0.94$ for daily data and $\lambda = 0.97$ for monthly data (Campbell, 2005).

Throughout this paper, stock market returns are defined as continuously compounded or log returns at time t , r_t , calculated as follows:

$$r_t = \log\left(\frac{P_t}{P_{t-1}}\right) = \ln P_t - \ln P_{t-1} \quad (4)$$

where P_t and P_{t-1} are the closing market indices at days t and $t-1$, respectively.

3.1 Unconditional coverage test

To assess the accuracy of the VaR model we need to implement back tests of model. In another word, an accurate VaR measure should satisfy both the unconditional coverage property and independent property. When we back testing the risk model, we construct a sequence of I_t across T days. The unconditional coverage property means that the probability of realization of a loss in excess of the estimated $VaR_t(\alpha)$ must be exactly $\alpha\%$ (i.e., $P(I_t(\alpha) = \alpha)$). If we want to test the fraction of exceptions for a particular risk model, call it π . In order to test it, we write the likelihood of an independent and identical distribution (i.i.d) Bernoulli sequence (Christoffersen, 2012):

$$L(\pi) = \prod_{i=1}^T (1 - \pi)^{1-I_t} \pi^{I_t} = (1 - \pi)^{T_0} \pi^{T_1} \quad (5)$$

Where T_0 and T_1 are the number of 0 and 1 in the sample ($T_0 + T_1 = T$). We can easily estimate π from $\hat{\pi} = T_1/T$

$$L(\hat{\pi}) = (T_0/T)^{T_0} (T_1/T)^{T_1} \quad (6)$$

Under the unconditional coverage null hypothesis that $\pi=p$, where p is the known VaR coverage rate, we have the likelihood

$$L(p) = (1 - p)^{T_0} p^{T_1} \quad (7)$$

We can check the unconditional coverage hypothesis using the likelihood ratio

$$LR_{uc} = -2 \log \left[\frac{L(p)}{L(\hat{\pi})} \right] \quad (8)$$

Asymptotically, the test will be a χ_1^2 with one degree of freedom.

Choosing a significance level of 5% for the test, we will have a critical value of 3.84 from the χ_1^2 distribution. If the LR_{uc} test value is larger than 3.84 then we reject the VaR model at 5% level. Alternatively, we can calculate the p -value associated with our test statistic.

The p -value is

$$p - value = 1 - F_{\chi_1^2}(LR_{uc}) \quad (9)$$

Where $F_{\chi_1^2}$ denotes the cumulative function of a χ_1^2 variable with one degree of freedom. If the p -value is below to the desired significance level, then we can reject the null hypothesis.

3.2 Independence Testing

If the VaR exceptions are clustered, then the risk manager can easily predict that if today is an exception, then tomorrow is more than p likely to be an exception as well. This is clearly not satisfactory. In such situation, the risk manager should increase the VaR in order to lower the conditional probability of an exception to the promised p (Christoffersen & Pelletier, 2004).

Our task is to establish a test that will be able to reject a VaR with clustered exceptions. To this end, assume the exception sequence is dependent over time and that it can be described as a first order Markov sequence with transition probability matrix:

$$\Pi_1 = \begin{bmatrix} \pi_{00} & \pi_{01} \\ \pi_{10} & \pi_{11} \end{bmatrix} \quad (10)$$

with π_{ij} the transition probability

$$\pi_{ij} = P[I_t = i \text{ and } I_{t+1} = j] \quad (11)$$

Note that $\pi_{00} = 1 - \pi_{01}$ and $\pi_{10} = 1 - \pi_{11}$

For example, π_{01} is the probability of an exception after a „non exception“, π_{11} is the probability of two consecutive exceptions.

If we observe a sample of T observations, then we can write the likelihood function of the first-order Markov process as:

$$L(\Pi_1) = \pi_{00}^{T_{00}} \pi_{01}^{T_{01}} \pi_{10}^{T_{10}} \pi_{11}^{T_{11}} \quad (12)$$

with T_{ij} is the number of observations with a j following an i .

Taking first derivatives with respect to π_{01} and π_{11} and setting these derivatives to zero, we can solve for the maximum likelihood estimate:

$$\hat{\pi}_{01} = \frac{T_{01}}{T_{00} + T_{01}} \quad (13)$$

$$\hat{\pi}_{11} = \frac{T_{11}}{T_{10} + T_{11}} \quad (14)$$

Note that, $T_{i0} + T_{i1} = T_i$, so that the estimated matrix is:

$$\hat{\Pi}_1 = \begin{bmatrix} T_{00}/T_0 & T_{01}/T_0 \\ T_{10}/T_1 & T_{11}/T_1 \end{bmatrix} \quad (15)$$

Allowing for dependence in the exception sequence corresponds to allowing π_{01} to be different from π_{11} . We are typically worried about positive dependence, which amounts to the probability of an exception following an exception (π_{11}) being larger than the probability of an exception following a non exception (π_{01}). If, on the other hand, the exceptions are independent over time, then the probability of an exception tomorrow does not depend on today being an exception or not, and we write

$$\pi_{01} = \pi_{11} = \pi \quad (16)$$

Under independence, the transition matrix is thus

$$\hat{\Pi} = \begin{bmatrix} 1 - \hat{\pi} & \hat{\pi} \\ 1 - \hat{\pi} & \hat{\pi} \end{bmatrix} \quad (17)$$

With $\hat{\pi} = T_1/T$

We can test the independence hypothesis that $\pi_{01} = \pi_{11}$ using a likelihood ratio test

$$LR_{ind} = -2 \log [L(\hat{\Pi}) / L(\hat{\Pi}_1)] \sim \chi_1^2 \quad (18)$$

Where $L(\hat{\Pi})$ is the likelihood under the alternative hypothesis from the LR_{uc} test.

3.3 Conditional Coverage Testing

Ultimately, we care about simultaneously testing if the VaR violations are independent and the average number of violations is correct. We can test jointly for independence and correct coverage using the conditional coverage test (Christoffersen, 2008)

$$LR_{cc} = -2 \log [L(p) / L(\hat{\Pi}_1)] \sim \chi_2^2 \quad (19)$$

Which corresponds to testing that $\pi_{01} = \pi_{11} = p$

Notice that $LR_{cc} = LR_{uc} + LR_{ind}$

4. Data and Empirical Analysis

In our research paper, we collect daily data of indices prices from official web sites of stock exchanges. Stock indices namely, are Belex 15, DJIA, SAX and STOXX Eastern Europe index. For each country, we use a total number of 1760 daily returns starting from December 31th, 2005 to December 31th, 2012 to estimate the initial parameters for VaR calculations using a 500-day rolling window. Because of the rolling-window methodology that we employ, our final sample includes 1260 estimated values of different VaR approaches for the period December 31th, 2007 – December 31th, 2012 which overlaps with the global financial crisis. All the descriptive and statistical tests, VaR calculations and back testing in this paper are performed in MS Excel and E Views software packages.

In order to deliver accurate risk predictions, the model should reflect the following stylized facts of daily asset returns:

- The expected daily returns have little or no predictability.
- The variance of daily returns greatly exceeds the mean.
- The variance of daily returns is predictable.
- Daily returns are not normally distributed.
- Even after standardizing daily returns by a dynamic variance model, the standardized daily returns are not normally distributed.
- Positive and negative returns of the same magnitude may have different impacts on the variance.
- Correlations between assets appear to be time-varying.

We test all above mentioned stylized facts of returns in tables which are presented hereinafter.

Table 1 gives the descriptive statistics for the Belex 15, DJIA, SAX and STOXX Eastern Europe index daily stock market log-returns. Results show that all stock markets average daily returns are not significantly different from zero (0). The highest standard deviation has the index STOXX TIM in the amount of 1.87% while the lowest value in the observed period record SAX (1.17%).

Large standard deviation compared to the mean supports the evidence that daily changes are dominated by randomness and small mean can be disregarded in risk measure estimates (Tsay, 2002).

The quartiles present in table (Q1, Q3) inscribe 50% of the values in the sample. The inter-quartile range (IQR) can be used to characterize the data when there may be extremities that skew the data. The interquartile range is a relatively robust statistic (also sometimes called "resistant") compared to the range and standard deviation.

Results for skewness show that three indices are negatively skewed (DJIA, STOXX TIM and SAX). Additional statistical tests based on hypothesis testing indicate that in case of DJIA skewness (-0.05) is not significantly different from zero (target value of skewness in testing normality) which mean that this index has symmetrical distribution. In the case of other three indices, the distribution has longer, fatter tails and higher probabilities for extreme events than for the normal distribution. Negative skewness implies that the distribution has a long left tail in cases of SAX and STOXX TIM. Belex 15 is the only one of the analyzed indices which recorded a significantly positive skewness.

High value of excess kurtosis dominated over this period as follows: SAX, Belex 15, STOXX TIM, and DJIA. According to that we have fat tails in all indices, with extremely high value in index SAX. For final check of normality distribution assumption we employed three different normality tests Jarque-Bera, Sharpio Wilk and Doornick Chi-Square test (Table 2).

Table 1: Descriptive statistics of indices 31.12.2005.-31.12.2012.

	Belex 15	DJIA	STOXX TIM	SAX
AVERAGE	-0,04%	0,01%	-0,01%	-0,05%
STD DEV	1,56%	1,34%	1,87%	1,17%
SKEW	0,17	-0,05	-0,36	-1,89
KURTOSIS	10,82	8,91	9,39	30,34
MEDIAN	-0,06%	0,05%	0,07%	0,00%
MIN	-10,9%	-8,20%	-13,58%	-14,8%
MAX	12,2%	10,51%	16,50%	11,9%
Q 1	-0,69%	-0,47%	-0,78%	-0,26%
Q 3	0,59%	0,57%	0,91%	0,31%

Source: Author's calculation

Table present normality tests and their p-value for each of the logarithmic daily returns on the four time series considered in this paper. Normality tests indicate that their unconditional distribution is not normal distributed. All tests strongly reject the null hypothesis of normality for all series. The null hypothesis of the Jarque-Bera test is rejected at the 0 per cent level for all indices, and the null hypothesis of other two tests also is rejected at the 0 per cent level for all series. The p-value is confidence level at which the null hypothesis is rejected. The hypotheses in these tests for normality are as follows: H_0 – indices returns are normally distributed; H_1 - indices returns are not normally distributed. (Tsay, 2002)

Table 2: Normality tests

	Belex 15	DJIA	STOXX TIM	SAX
*SL = 5%	p-value	p-value	p-value	p-value
Jarque-Bera	0,0%	0,0%	0,0%	0,0%
Shapiro-Wilk	0,0%	0,0%	0,0%	0,0%
Doomick Chi-Square	0,0%	0,0%	0,0%	0,0%

Note: * denote significance level at the 5%

Source: Author’s calculation

When there is a strong deviation from normality, correlation analysis should be done. Large number of financial time series data sets exhibit time interdependency among their value. This is very important to detect potential presence of autocorrelation in analyzed stock markets, because it can help us to improve the forecast quality of the risk model. In that sense, we have done two separately tests: Ljung-Box (white noise) and Engle ARCH tests to examining the empirical full period, crossing from 12/31/2005 through 12/31/2012. The results are shown in table 3.

Table 3: White-noise and ARCH effect tests

	Test	p-value	5%
BELEX 15	White-noise	0,00%	FALSE
	ARCH Effect	0,00%	TRUE
DJIA	White-noise	0,00%	FALSE
	ARCH Effect	0,00%	TRUE
STOXX TIM	White-noise	0,00%	FALSE
	ARCH Effect	0,00%	TRUE
SAX	White-noise	0,77%	FALSE
	ARCH Effect	99,95%	FALSE

Source: Author’s calculation

Based on results of white-noise test from table, we answer the question: Does log-returns time series exhibit white-noise (no serial correlation). In all observed indices answer for our question is no, i.e. the daily log returns have a presence of serial correlation.

According to table 1, where we concluded that the daily log returns distribution possess a fat-tails (excess kurtosis > 0) guide us to assumption that squared returns are correlated. In order to examine that phenomenon, we employed ARCH effect test, which testing autocorrelation of squared log returns. Autoregressive conditional heteroskedasticity (ARCH) models were developed by Engle who also proposed a test that explicitly examines for non-linearity in the second moment, i.e. the Engle test examines evidence for non-linearity in the variance(Engle, 1982). In sum, the ARCH test helps us to detect a time varying phenomenon in the conditional volatility, and thus suggests different types of models (e.g. ARCH/GARCH) to capture these dynamics.

Examining the ARCH effects test results from table 3, we conclude that the squared returns are serially correlated, i.e. we have a conditional heteroskedasticity in the three analyzed indices daily log returns, except SAX index, where we do not have ARCH effect.

We evaluate the risk models in the context of daily returns on four equity price indices for which a relatively long history is available for analysis. VaR is estimated based on revised parameter estimates (or percentiles) using the sample of 500 days. As the maximum sample size for estimation of parameters is 500 days, the first VaR estimation occurs 500

days (nearly 2 years) into the data sample. In the case of the all indices this allows us to calculate 1260 estimates of VaR having a 1-day horizon. Hence, data sets for VaR estimation starts 500 days later and cover the period from 31/12/2007 to 31/12/2012, in order to initialize the estimation. We include exponentially weighted moving average approach ($\lambda=0.94$) and historical simulation approaches (500 days rolling window) to calculate one day ahead VaR for two given different confidence levels (95%, 99%).

In table 4 we calculate average 1-day VaR for both models at 95% and 99% confidence level for all indices during time horizon from 12/21/2007 to 12/31/2012.

Table 4: average 1-day Vary from 12/31/2007 to 12/31/2012

	RiskMetrics		Historical Simulation	
	1-day VaR		1-day VaR	
	1,00%	5,00%	1,00%	5,00%
Belex 15	3,34%	2,36%	4,74%	2,44%
DJIA	3,00%	2,12%	4,15%	2,34%
STOXX TIM	4,09%	2,89%	5,99%	3,12%
SAX	2,67%	1,89%	4,47%	1,78%

Source: Author’s calculation

Through a simulation methodology, we attempt to determine how each VaR approach would have performed over a realistic range of indices over the sample period. In our empirical analysis, we investigate both in-sample (training) performance and out-of-sample (forecasting) performance based on the Unconditional coverage test, Christoffersen independent test and Conditional coverage test. Full back testing diagnostics of 1260 VaR forecasts based on first 500 days observations from beginning of sample period for analyzed stock indices, at 95% and 99% confidence level are presented in table 5. In detailed calculations steps we checked the unconditional coverage, independent and conditional coverage hypothesis using likelihood ratio test, which have a χ_1^2 distribution with one degree of freedom and χ_1^2 distribution with two degree of freedom for conditional coverage test. We used a significance level of 5% for the tests, where a critical value from the χ_1^2 distribution with one degree of freedom is 3.84. If the $LR_{uc,ind}$ tests value are larger than 3.84 then we reject the VaR model at 5% level, otherwise we accept VaR model. In the case of conditional coverage test, critical value from the χ_1^2 distribution is 5,99.

For historical simulation there are two rejections in the VaR 99%, while three VaR 95% (SAX, STOXX and Belex 15) passed unconditional coverage tests.

For Risk Metrics number of rejections in the VaR 99% is three and two in VaR 95%. It is interesting that Belex 15 and SAX passed unconditional coverage tests for both historical simulation and Risk Metrics approach at 95% and 99% confidence level. The poor backtesting performance of DJIA and STOXX during backtesting period can be explained with negative effects of financial crisis that is reflected in high volatile of equity market.

Results of independence tests show that Risk Metrics performs better than historical simulation. Accordingly property of independent, in three analyzed indices (DJIA, SAX and STOXX TIM) Risk Metrics VaR 95% and 99% accurately capture market risk. Belex 15 failed both VaR (95% and 99%) models. The reason for these results should be found in potential clustering of volatility in given stock markets, which leads us to conclusion that VaR exceptions also cluster in time and are not independent.

Table 5: Backtesting results and diagnostics of 1260 VaR forecasts for analyzed indices daily log returns, 95% and 99% confidence level, from 12/31/2007 to 12/31/2012

	SAX				DJIA				STOXX				BELEX 15			
	RM		HS													
	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%
T_0	1224	1191	1242	1185	1231	1178	1236	1176	1233	1180	1239	1182	1242	1202	1243	1208
T_1	36	69	18	75	29	82	24	84	27	80	21	78	18	58	17	52
T_{00}	1189	1125	1224	1120	1203	1099	1214	1102	1208	1109	1220	1121	1228	1155	1232	1172
T_{01}	35	66	18	65	28	79	22	74	25	71	19	61	14	47	11	36
T_{10}	35	66	18	65	28	79	22	74	25	71	19	61	14	47	11	36
T_{11}	1	3	0	10	1	3	2	10	2	9	2	17	4	11	6	16
\hat{p}	2,86%	5,48%	1,43%	5,95%	2,30%	6,51%	1,90%	6,67%	2,14%	6,35%	1,67%	6,19%	1,43%	4,60%	1,35%	4,13%
$L(p)$	1,01E-71	6,68E-117	1,06E-41	3,30E-124	1,12E-60	1,88E-132	2,47E-52	9,37E-135	2,17E-57	3,99E-130	4,12E-47	8,92E-128	1,06E-41	7,20E-103	7,56E-40	7,98E-95

L(p)	4,54E-78	4,99E-117	3,79E-42	1,06E-124	4,24E-64	1,19E-133	4,03E-54	3,28E-136	4,15E-60	4,28E-131	3,91E-48	1,55E-128	3,79E-42	5,81E-103	3,75E-40	2,73E-95
p ₀₁	2,86%	5,54%	1,45%	5,49%	2,27%	6,71%	1,78%	6,29%	2,03%	6,02%	1,53%	5,16%	1,13%	3,91%	0,88%	2,98%
p ₁₁	2,78%	4,35%	0,00%	13,33%	3,45%	3,66%	8,33%	11,90%	7,41%	11,25%	9,52%	21,79%	22,22%	18,97%	35,29%	30,77%
L(p ₁)	1,01E-71	7,35E-117	0,00E+00	6,56E-123	1,21E-60	3,72E-132	0,00E+00	4,92E-134	6,79E-57	1,69E-129	0,00E+00	8,29E-123	3,48E-38	3,90E-99	7,36E-33	5,42E-85
LR _{uc}	29,229	0,584	2,064	2,274	15,765	5,531	8,234	6,701	12,522	4,465	4,711	3,506	2,064	0,429	1,399	2,144
LR _{ind}	0,001	0,192	0	5,978	0,151	1,363	0	3,317	2,276	2,887	0	22,879	16,185	17,194	32,182	45,277
LR _{cc}	29,23	0,776	0	8,252	15,917	6,894	0	10,019	14,798	7,352	0	26,386	18,248	17,623	33,582	47,421
Hypothesis Testing (Chi-Square Test) -5% significant level																
LR _{uc}	Reject	Accept	Accept	Accept	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Accept	Accept	Accept	Accept	Accept
LR _{ind}	Accept	Accept	Accept	Reject	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Reject	Reject	Reject	Reject	Reject
LR _{cc}	Reject	Accept	Accept	Reject	Reject	Reject	Accept	Reject	Reject	Reject	Accept	Reject	Reject	Reject	Reject	Reject

According to conditional coverage test, which combines property of unconditional coverage and independence, HS approach seem to do better job than RM approach. Results indicate that HS VaR 99% accurately measure and forecast market risk in three analyzed equity markets as follows: SAX, DJIA and STOXX. It is very interesting that there is a difference between the results obtained by applying these models, between Serbian and Slovak capital markets.

Basically, both countries are characterized by inefficiency of capital markets. The cause of inefficiency is the lack of quality stocks that are quoted on the stock exchange, liquidity, market depth and breadth as well as low volume trading.

As final criteria for validation of VaR models we choose results of first two tests and based our findings on the fact that VaR models must satisfy both unconditional coverage and independence property. According that, since neither one of the tested models did not fully satisfy both criteria of back testing VaR models, except VaR 99% HS and VaR 95% RM approach in the case of SAX index, which even fulfill all three back tests, we can conclude that all the applied models significantly underestimates the true level of market risk in observed window length. At the end of our back testing procedure, we observe that different models are best suited to different indices and thus cannot identify a single model working properly for all four indices.

5. Conclusion

This research paper examined the relative performance of Value-at-Risk (VaR) as market risk measure in developed and emerging countries by using unconditional coverage, independence and conditional coverage tests. Data sample includes daily stock indices returns from Serbia, Slovakia, USA and international portfolio, which consists of emerging market stocks, from 2005 to 2012. All analyzed equity markets in observed window length have become increasingly volatile and characterized by unusually high volatility levels and huge losses. The only exception is SAX index.

As a result of this event VaR models that are applied in the work do not properly predict market risk, even in the most efficient capital markets such as the US stock market. We can conclude that performance of VaR was worse for developed than for emerging countries during the global financial crisis. This situation can be explained by the fact that the developed countries have been affected from the crisis more than emerging countries.

Additional reason for poor performance of VaR models in Belex and STOXX is the European debt crisis, which is currently present in most of the countries in the Eurozone, and it also has direct implications in Serbian financial system and market.

Based on our results and findings we can also conclude that there is not benefit of international diversification on accurate relative performance of VaR measure in case of STOXX Eastern Europe Total Market Index (TMI), as represent of international portfolio of European emerging stock markets.

In case of Serbian stock market the situation is more complex in field of market risk measurement because returns of index Belex 15 are characterized by significant autocorrelation and heteroskedasticity, which considerably complicates VaR estimation. It is very interesting that there is a difference between the results obtained by applying these models, between Serbian and Slovak capital markets. By author's opinion, this situation can be explained by the fact that the Slovak capital market is less volatile than Serbian. Positive results from the application of the VaR model on Slovakia stock market can be explained by the relatively stable movement of the index SAX. In the period before the crisis, one of the features of this index was that its value is not rapidly and constantly growing, as in the case of Belex 15. The global financial crisis that occurred did not significantly influence on decline in value of the index. This is one more reason why use of VaR models on the Slovak stock market gives much better results than Serbian.

Since none of the tested models fully satisfies both criteria of back testing VaR models, except VaR 99% HS and VaR 95% RM approach in the case of SAX index, we can conclude that all the applied models significantly underestimate market risk during observed period in analyzed stock markets. This result is consistent with the poor performance of VaR at some financial institutions in 2008. We consider that for the relevant assessment of market risk exposure in capital markets in turbulent times, it is preferable to apply more complex and sophisticated measures of risk such as VaR

methods based on conditional volatility with different assumptions of innovation distribution (GARCH-based). Also need to mention that the characteristic of stock markets and their asset returns in combination with the desired confidence level and risk horizon determine how well a certain approach performs on a certain financial market.

References

- Alexander, C. (2008). *Value at Risk Models*. Chichester: John Wiley & Sons.
- Anatolyev, S. (2006). *Nonparametric retrospection and monitoring of predictability of financial returns*. Moscow: Centre for Economic and Financial Research at New Economics School.
- Avdulaj, K. (2010). *VaR based on extreme Value Theory Methods and copulas. Empirical evidence from Central Europe*. Prague: Charles University in Prague.
- Badić, P. (2005). Use VaR methods for measuring market risk and calculating capital adequacy. *Biometrika, Volume XIII*, 3.
- Barjaktarović L., & Ječmenica, D. (2011). Optimism vs. Pessimism of competitiveness of the banking sector of Serbia. *Industrija*, 39(2), 137-150.
- Barjaktarović L., Filipović S. & Dimić M. (2013). Concentration level of the banking industry in CEE countries. *Industrija*, 41 (3), 39-53.
- Campbell, S. (2005). *A review of backtesting and backtesting procedures*. Washington: Finance and Economics Series, federal Reserve Board.
- Christoffersen, P. (2012). *Elements of financial risk management*. San Diego: Academic press.
- Christoffersen, P., & Pelletier, D. (2004). Backtesting Value at Risk: A duration Based Approach. *Journal of Financial Econometrics* 2, 84-108.
- Christoffersen, P., & Peter, F. (2008). *Backtesting, prepared for the Encyclopedia of Quantitative Finance*. Chichester: John Wiley & Sons.
- Dowd, K. (2002). *Measuring Market Risk*. Chichester: John Wiley & Sons.
- Duffie, D., & Pan, J. (1997). An overview of Value at Risk. *The Journal of Derivatives*, 7-49.
- Engle, F. (1982). Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation. *Econometrica*, 987-1007.
- Ergen, I. (2010). *Essays in financial risk management*. Houston: University of Texas.
- Garbade, K. D. (1987). *Assessing and allocation interest rate risk for a multisector bond portfolio consolidated over multiple profit centers*. New York: Bankers Trust.
- Guldimann, T. (2000). The story of Risk Metrics. *Risk*, 56-58.
- Hardy, C. O. (1923). *Risk and Risk-Bearing*. Chicago: University of Chicago.
- Hicks, J. (1935). A suggestion for simplifying the theory of money. *Economica*, 1-19.
- Jorion, P. (2002). *Value at Risk: the new benchmark for managing financial risk*. New York: McGraw-Hill.
- Jurun, E., Pivac, S., & Arnerić, J. (2007). Historical and Prognostic Risk Measuring Across Stocks and Markets. *Journal of WSEAS Transactions on Business and Economics*, 126-134.
- Khindanova, I., & Rachev, S. (2000). *Value at Risk: Recent Advances. Handbook of Analytic Computational Methods in Applied Mathematics*. New York: Chapman & Hall/CRS.
- Klieštik, T. (2011). Value at Risk Methods. *Logistyk-nauka*, 1645-1649.
- Kovačić, Z. (2007). *Forecasting volatility: Evidence from Macedonian stock exchange*. Skopje: MPRA Paper No 5319.
- Kupiec, P. (1995). Test statistics for verifying the accuracy of risk management models. *Journal of Derivatives*, 73-84.
- Leavens, D. H. (1945). Diversification of investments. *Trusts and Estates*, 469-473.
- Lintner, J. (1965). The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. *Review of Economics and Statistics*, 13-37.
- Markowitz, H. M. (1952). Portfolio Selection. *Journal of Finance*, 77-91.
- Markowitz, H. M. (1959). *Portfolio Selection: Efficient Diversification of Investments*. New York: John Wiley & Sons.
- Marshall, C., & Sigel, M. (1997). Value at Risk: Implementing a Risk measurement standard. *The Journal of Derivatives*, 91-111.
- Mossin, J. (1966). Equilibrium in capital asset market. *Econometrica*, 768-783.
- Neftçi, S. (2000). Value at Risk Calculations, Extreme Events and Tail Estimation. *The Journal of Derivatives*, 23-37.
- Radiojević, N., Milojković, D., & Stojković, D. (2010). Testing the Applicability of parametric and nonparametric methods of Value at Risk at Serbian Capital Market. *Bankarstvo*, 46-61.
- Roy, A. D. (1952). Safety first and the holding of asset. *Econometrica*, 431-449.
- Sharp, W. F. (1964). A simplified model for portfolio analysis. *Management Science*, 277-293.
- Taleb, N. (1997). *Dynamic Hedging*. New York: John Wiley & Sons.
- Terzić, I., & Milojević, M. (2013). Evaluating measures of market risk in circumstances of global financial crisis – empirical evidence from five countries. CBU International Conference Proceedings (stránky 75-86). Prague: Central Bohemia University.
- Tobin, J. (1958). Liquidity preference as behavior towards risk. *The Review of Economic Studies*, 65-86.
- Treynor, J. (1961). *Towards a theory of market value of risky assets*. Unpublished manuscript.
- Tsay, R. (2002). *Analysis of financial time series*. New York: JW&S.
- Wilson, T. (1993). Infinite wisdom. *Risk*, 37-45.
- Živković, S. (2006). Applying Hybrid approach to calculate VaR in Croatia. *Proceedings of the International Conference From Transition to Sustainable Development: The path to European Integration* (pp. 50-71). Sarajevo: Faculty of Economics in Sarajevo.
- Živković, S. (2007). Testing popular VaR models in EU new members and candidates states. *Journal of Economics and Business, Rijeka Faculty of Economics*.