Synthesis of Four-Link Basic Kinematic Chains [BKC] with Spherical Pairs for Spatial Mechanisms

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Abstract

A solution to the problem of synthesizing an initial three-dimensional kinematic chain with spherical and rotary kinematic pairs is presented. It is shown that this chain can be used as a structural module for structural-kinematic synthesis of three-dimensional four-link motion generating lever mechanisms by the preset positions of the in-and output links.

Keywords: *mechanism, four-link, kinematic pairs, kinematic chains, synthesis*

1. Introduction

These papers demonstrate that four-link basic kinematic chains (BKC) may be used as a structural module with structural and kinematic synthesis of plain linkage mechanisms. Such an approach to the synthesis of plain mechanisms allows reducing the problem of their structural and kinematic synthesis to the solution of problem of BKC synthesis, which is very useful for automation of mechanisms engineering. This paper testifies that specified approach may be applied to the problem of structural and kinematic synthesis of spatial linkage mechanisms. The solution of the problem of synthesis of spatial BKC of RSS type (R- rotational, S - spherical kinematic pairs) is represented and its use as a structural module with structural and kinematic synthesis of spatial linkage mechanisms as per predetermined positions of input and output links is shown. A method of solution the problem of BKC synthesis of RSS type is based on the introduction of two movable bodies invariably associated with the input and output links.

2. Theory

If two adjacent elements of open four-link BKC with spherical kinematic pairs are tending to infinity, then it is necessary to replace the spherical kinematic pair for the plain or cylindrical.

Proof: If in formula (6) $D_1 = 0$, then $(X_A, Y_A, Z_A) \to \infty$ and center of circumference approaching sphere will be laid in the plane or along a straight line. Then, in addition to the required parameters of BKC $B(x_B, y_B, z_B) \in Q_1$ and $C(x_C, y_C, z_C) \in Q_2$ with common parameter R, on fixed system of coordinates $OXYZ$ instead of point $A(X_A, Y_A, Z_A) \in Q$ it is necessary to determine coefficients a, b, c, of plane Q.

This section considers a problem of synthesis of spatial basic kinematic chains (BKC) with rotating, plane and spherical kinematic pairs as per specified positions of input and output links based on introduction of two movable solids all the time connected with input and output links[1].

Problem statement: Given N of finite distant positions of two solids Q_1 и Q_2 : $Q_1(x_A, y_A, z_A, \psi_{1i}, \theta_{1i}, \varphi_{1i})$,

$Q_2(x_{Di}, y_{Di}, z_{Di}, \psi_{2i}, \theta_{2i}, \varphi_{2i})$, $(i = \overline{1,N})$,

where, ψ_{ji}, θ_{ji}, φ_{ji} - fixed axis Eulerian angles $OXYZ$ (Figure 1).

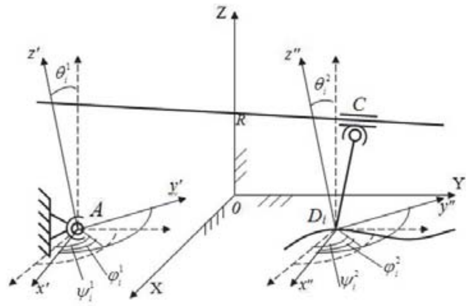

Fig .1 Equivalent four-link kinematic chain ACD.

It is required to find a point $A(x_A, y_A, z_A)$ on solid Q, to find plane on solid Q_1

$$ax + by + cz + d = 0 \tag{1}$$

and point $C(x_C, y_C, z_C)$ on solid Q_1, which in its motion about solid Q_1 approached to desired plane (1). Equation of plane (1) on fixed solid Q is determined by known transformation formulas

$$a[(X_{Ci} - X_A)t_{11} + (Y_{Ci} - Y_A)t_{21} + (Z_{Ci} - Z_A)t_{31}] +$$
$$+ b[(X_{Ci} - X_A)t_{12} + (Y_{Ci} - Y_A)t_{22} + (Z_{Ci} - Z_A)t_{32}] +$$
$$+ c[(X_{Ci} - X_A)t_{13} + (Y_{Ci} - Y_A)t_{23} + (Z_{Ci} - Z_A)t_{33}] + d = 0 \tag{2}$$

where, ψ_{ji}, φ_{ji} - angles are given, and angle $\theta_{ji} = 0$, $j = 1.2$, $i = \overline{1,N}$.

$$\alpha_{ji} = \psi_{1i} + \varphi_{1i}, \beta_{ji} = \psi_{2i} + \varphi_{2i}$$

$$\begin{bmatrix} X_{Ci} \\ Y_{Ci} \\ Z_{Ci} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\psi_{2i} + \varphi_{2i}) & -\sin(\psi_{2i} + \varphi_{2i}) & 0 & X_{Di} \\ \sin(\psi_{2i} + \varphi_{2i}) & \cos(\psi_{2i} + \varphi_{2i}) & 0 & Y_{Di} \\ 0 & 0 & 1 & Z_{Di} \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_C \\ y_C \\ z_C \\ 1 \end{bmatrix} \tag{3}$$

$$t_{11} = \cos(\psi_{1i} + \varphi_{1i}),\ t_{21} = \sin(\psi_{1i} + \varphi_{1i}),\ t_{12} = -\sin(\psi_{1i} + \varphi_{1i}),$$
$$t_{22} = \cos(\psi_{1i} + \varphi_{1i}),\ t_{33} = 1,\ t_{13} = t_{31} = t_{23} = t_{32} = t_{41} = t_{42} = t_{43} = 0.$$

After substitution of the expression (3) in formula (2) and required transformations let us comprise the weighted difference Δq_i of point $C_i(x_C, y_C, z_C)$ from plane (2) as:

$$\Delta q_i = G_1 \cos\alpha_{ji} + G_2 \sin\alpha_{ji} + G_3 \cos(\alpha_{ji} - \beta_{ji}) + G_4 \sin(\alpha_{ji} - \beta_{ji}) + G_5 X_i +$$
$$+ G_6 Y_i + G_7 Z_i + G_8 + G_9 - 1 \tag{4}$$

where,

$$G_1 = -(aX_A + bY_A),\ G_2 = bX_A - aY_A,\ G_3 = ax_C + by_C,\ G_4 = ay_C - bx_C,$$
$$G_5 = a,\ G_6 = b,\ G_7 = c,\ G_8 = cz_C,\ G_9 = -cZ_A.$$

$$\begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix} = \begin{bmatrix} \cos\alpha_{ji} & \sin\alpha_{ji} & 0 \\ -\sin\alpha_{ji} & \cos\alpha_{ji} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} X_{Di} \\ Y_{Di} \\ Z_{Di} \end{bmatrix}$$

It should be noted that ten required parameters enter into the expression (2), but after normalization of straight-line equation $d = -1$, the nine required parameters remain. These are coefficients of equation of plane a, b, c, coordinates X_A, Y_A, Z_A, points $A \in Q$ and coordinates x_C, y_C, z_C, points $C \in Q_2$.

Let us comprise sum of squares of weighted difference for N positions

$$S = \sum_{i=1}^{N} [\Delta q_i]^2 \quad (i = \overline{1,N}) \tag{5}$$

Three systems of equations are obtained, the solution of which is written in the following form of relative required parameters[2]:

$$(X_A, Y_A, Z_A) = \frac{1}{D_1}(D_{XA}, D_{YA}, D_{ZA}) \quad \text{(where } D_1 \neq 0\text{)} \tag{6}$$

Stationary conditions per variables

$$\frac{\partial S}{\partial j} = 0 \quad (j = G_1, G_2, G_3, G_4, G_5, G_6, G_7, G_8, G_9);$$

would result in the following simultaneous linear algebraic equations as $G_1 \div G_9$;

$$A \cdot \overline{G} = \overline{B} \tag{7}$$

where, matrix elements A (9,9):

$a_{11} = \sum \cos^2 \alpha_{ji}$, $a_{12} = a_{21} = \frac{1}{2}\sum \sin \alpha_{ji} \cos \alpha_{ji}$, $a_{13} = a_{31} = \sum \cos \alpha_{ji} \cdot \cos(\alpha_{ji} - \beta_{ji})$,
$a_{14} = a_{41} = \sum \cos \alpha_{ji} \cdot \sin(\alpha_{ji} - \beta_{ji})$, $a_{15} = a_{51} = \sum X_i \cdot \cos \alpha_{ji}$, $a_{16} = a_{61} = \sum Y_i \cdot \cos \alpha_{ji}$,
$a_{17} = a_{71} = \sum Z_i \cdot \cos \alpha_{ji}$, $a_{18} = a_{81} = \sum \cos \alpha_{ji}$, $a_{19} = a_{91} = \sum \cos \alpha_{ji}$, $a_{22} = \sum \sin^2 \alpha_{ji}$,
$a_{23} = a_{32} = \sum \sin \alpha_{ji} \cdot \cos(\alpha_{ji} - \beta_{ji})$, $a_{24} = a_{42} = \sum \sin \alpha_{ji} \cdot \sin(\alpha_{ji} - \beta_{ji})$,
$a_{25} = a_{52} = \sum X_i \cdot \sin \alpha_{ji}$, $a_{26} = a_{62} = \sum Y_i \cdot \sin \alpha_{ji}$, $a_{27} = a_{72} = \sum Z_i \cdot \sin \alpha_{ji}$,
$a_{28} = a_{82} = \sum \sin \alpha_{ji}$, $a_{29} = a_{92} = \sum \sin \alpha_{ji}$, $a_{33} = \sum \cos^2(\alpha_{ji} - \beta_{ji})$,
$a_{34} = a_{43} = \frac{1}{2}\sum \sin(\alpha_{ji} - \beta_{ji}) \cdot \cos(\alpha_{ji} - \beta_{ji})$, $a_{35} = a_{53} = \sum X_i \cdot \cos(\alpha_{ji} - \beta_{ji})$,
$a_{36} = a_{63} = \sum Y_i \cdot \cos(\alpha_{ji} - \beta_{ji})$, $a_{37} = a_{73} = \sum Z_i \cdot \cos(\alpha_{ji} - \beta_{ji})$,
$a_{38} = a_{83} = \sum \cos(\alpha_{ji} - \beta_{ji})$ $a_{39} = a_{93} = \sum \cos(\alpha_{ji} - \beta_{ji})$, $a_{44} = \sum \sin^2(\alpha_{ji} - \beta_{ji})$,
$a_{45} = a_{54} = \sum X_i \cdot \sin(\alpha_{ji} - \beta_{ji})$, $a_{46} = a_{64} = \sum Y_i \cdot \sin(\alpha_{ji} - \beta_{ji})$
$a_{47} = a_{74} = \sum Z_i \cdot \sin(\alpha_{ji} - \beta_{ji})$, $a_{48} = a_{84} = a_{49} = a_{94} = \sum \sin(\alpha_{ji} - \beta_{ji})$
$a_{55} = \sum X_i^2$, $a_{66} = \sum Y_i^2$, $a_{77} = \sum Z_i^2$, $a_{88} = a_{99} = N$, $a_{56} = a_{65} = \sum X_i^2$,
$a_{57} = a_{75} = \sum X_i Z_i$, $a_{58} = a_{85} = a_{59} = a_{95} = \sum X_i$, $a_{67} = a_{76} = \sum Y_i Z_i$,
$a_{68} = a_{86} = a_{69} = a_{96} = \sum Y_i$, $a_{78} = a_{79} = a_{87} = a_{97} = \sum Z_i$, $a_{89} = a_{98} = 1$,

$$\overline{G} = [G_1, ..., G_9]^T$$

$$\overline{B} = \left[\sum \cos\alpha_{ji}, \sum \sin \alpha_{ji}, \sum \cos(\alpha_{ji} - \beta_{ji}), \sum \sin(\alpha_{ji} - \beta_{ji}), \sum X_i, \sum Y_i, \sum Z_i, N, N\right]^T$$

Solution of system (5) enables you to determine the required parameters of synthesis. When coordinates of stand $D(X_D, Y_D, Z_D)$ of output link of slotted link mechanism have fixed values, you can determine nine required parameters of synthesis:

$$X_A = -\frac{aG_1 - bG_2}{a^2 + b^2}, \quad Y_A = -\frac{bG_1 + aG_2}{a^2 + b^2}, \quad Z_A = -\frac{G_9}{G_7}, \quad x_C = \frac{aG_3 - bG_4}{a^2 + b^2},$$

$$y_C = \frac{aG_4 + bG_3}{a^2 + b^2}, \quad z_C = \frac{G_8}{G_7}, \quad a = G_5, \quad b = G_6, \quad c = G_7, \quad a^2 + b^2 \neq 0, \quad c \neq 0.$$

Therefore, as per assigned positions of input and output links of transfer mechanisms, you can synthesize the spatial slotted link mechanisms of type RP_LS (R – rotational, P_L – plane, S - spherical kinematic pairs).

Now, let us consider a matter of choice of normalization of coefficients of equation of plane a, b, c. With normalization of $d = -1$, we obtain the weighted difference (3). When entering $a = -1$, $b = -1$, $c = -1$ into expression (3), we obtain exact expressions of displacements $(\Delta_i)_x$, $(\Delta_i)_y$, $(\Delta_i)_z$ of points C_i along axes OX, OY, OZ, respectively, which are weighted relative to displacement Δ_i along normal line. Therefore, you may not say beforehand which normalization is the best and so it would be reasonable to consider all four events.

1. Let us assume that $a=-1$. Then weighted difference considering (4) is as follows

$$\Delta q_i = G_1\cos\alpha_{ji} + G_2\sin\alpha_{ji} + G_3\cos(\alpha_{ji}-\beta_{ji}) + G_4\sin(\alpha_{ji}-\beta_{ji}) + \\ + G_5Y_i + G_6Z_i + G_7 + G_8 + G_9 - X_i \tag{8}$$

where,

$G_1 = X_A - bY_A$, $G_2 = Y_A + bX_A$, $G_3 = -x_C + by_C$, $G_4 = -(bx_C + y_C)$,
$G_5 = b$, $G_6 = c$, $G_7 = cz_C$, $G_8 = cZ_A$, $G_9 = d.$

Based on stationary state condition $S=\sum[\Delta q_i]^2$

$$\frac{dS}{dj}=0, \qquad (j = G_1 \div G_9) \tag{9}$$

considering (6) we obtain the following linear system:

$$A\cdot\overline{G} = \overline{B} \tag{10}$$

where, matrix elements A (9,9):

$a_{11} = \sum\cos^2\alpha_{ji}$, $a_{12} = a_{21} = \frac{1}{2}\sum\sin\alpha_{ji}\cos\alpha_{ji}$, $a_{13} = a_{31} = \sum\cos\alpha_{ji}\cdot\cos(\alpha_{ji}-\beta_{ji})$,
$a_{14} = a_{41} = \sum\cos\alpha_{ji}\cdot\sin(\alpha_{ji}-\beta_{ji})$, $a_{15} = a_{51} = \sum Y_i\cdot\cos\alpha_{ji}$, $a_{16} = a_{61} = \sum Z_i\cdot\cos\alpha_{ji}$,
$a_{17} = a_{71} = \sum\cos\alpha_{ji}$, $a_{18} = a_{81} = \sum\cos\alpha_{ji}$, $a_{19} = a_{91} = \sum\cos\alpha_{ji}$, $a_{22} = \sum\sin^2\alpha_{ji}$,
$a_{23} = a_{32} = \sum\sin\alpha_{ji}\cdot\cos(\alpha_{ji}-\beta_{ji})$, $a_{24} = a_{42} = \sum\sin\alpha_{ji}\cdot\sin(\alpha_{ji}-\beta_{ji})$,
$a_{25} = a_{52} = \sum Y_i\cdot\sin\alpha_{ji}$, $a_{26} = a_{62} = \sum Z_i\cdot\sin\alpha_{ji}$, $a_{27} = a_{72} = \sum\sin\alpha_{ji}$,
$a_{28} = a_{82} = \sum\sin\alpha_{ji}$, $a_{29} = a_{92} = \sum\sin\alpha_{ji}$, $a_{33} = \sum\cos^2(\alpha_{ji}-\beta_{ji})$,
$a_{34} = a_{43} = \frac{1}{2}\sum\sin(\alpha_{ji}-\beta_{ji})\cdot\cos(\alpha_{ji}-\beta_{ji})$, $a_{35} = a_{53} = \sum Y_i\cdot\sin(\alpha_{ji}-\beta_{ji})$,
$a_{36} = a_{63} = \sum Z_i\cdot\cos(\alpha_{ji}-\beta_{ji})$, $a_{37} = a_{73} = \sum\cos\alpha_{ji}(\alpha_{ji}-\beta_{ji})$, $a_{38} = a_{83} = \sum\cos(\alpha_{ji}-\beta_{ji})$ $a_{39} = a_{93} = \sum\cos(\alpha_{ji}-\beta_{ji})$,
$a_{44} = \sum\sin^2(\alpha_{ji}-\beta_{ji})$,
$a_{45} = a_{54} = \sum Y_i\cdot\sin(\alpha_{ji}-\beta_{ji})$, $a_{46} = a_{64} = \sum Z_i\cdot\sin(\alpha_{ji}-\beta_{ji})$
$a_{47} = a_{74} = \sum\sin(\alpha_{ji}-\beta_{ji})$, $a_{48} = a_{84} = \sum\sin(\alpha_{ji}-\beta_{ji})$, $a_{49} = a_{94} = \sum\sin(\alpha_{ji}-\beta_{ji})$,
$a_{55} = \sum Y_i^2$, $a_{56} = a_{65} = \sum Y_iZ_i$, $a_{57} = a_{75} = \sum Y_i$, $a_{58} = a_{85} = \sum Y_i$, $a_{59} = a_{95} = \sum Y_i$,
$a_{66} = \sum Z_i^2$, $a_{67} = a_{76} = \sum Z_i$, $a_{68} = a_{86} = \sum Z_i$, $a_{69} = a_{96} = \sum Z_i$,
$a_{77} = a_{88} = a_{99} = N$, $a_{78} = a_{87} = a_{79} = a_{97} = a_{89} = a_{98} = 1$,

$$X = [G_1, G_2, G_3, G_4, G_5, G_6, G_7, G_8, G_9]^T,$$

$$\overline{B} = [\sum X_i\cos\alpha_{ji}, \sum X_i\sin\alpha_{ji}, \sum X_i\cos(\alpha_{ji}-\beta_{ji}), \sum X_i\sin(\alpha_{ji}-\beta_{ji}), \\ \sum X_iY_i, \sum X_iZ_i, \sum X_i, \sum Y_i, \sum Z_i]^T$$

Solution of system (8) enables you to determine the required parameters of synthesis as:

$X_A = \dfrac{G_1 + bG_2}{1+b^2}$, $Y_A = \dfrac{G_2 - bG_1}{1+b^2}$, $Z_A = -\dfrac{G_8}{G_6}$, $x_C = -\dfrac{G_3 - bG_4}{1+b^2}$,
$y_C = \dfrac{-G_4 + bG_3}{1+b^2}$, $z_C = \dfrac{G_7}{G_6}$, $b = G_5$, $c = G_6$, $d = G_9$.

As a consequence of solution of this problem, we determine such coordinates of points $A(X_A, Y_A, Z_A)\in Q$, $C(x_C, y_C, z_C)\in Q_2$, and plane on solid Q_1 that by combining plane and spherical kinematic pairs with them, we obtain open-link spatial kinematic chain ACD of type RP_LS (R – rotational, P_L – plane, S - spherical kinematic pairs).

2. Let us assume that $b=-1$. Then weighted difference considering (4) is as follows

$$\Delta q_i = G_1 \cos\alpha_{ji} + G_2 \sin\alpha_{ji} + G_3 \cos(\alpha_{ji} - \beta_{ji}) + G_4 \sin(\alpha_{ji} - \beta_{ji}) +$$
$$+ G_5 X_i + G_6 Z_i + G_7 + G_8 + G_9 - Y_i ,$$

where,

$G_1 = Y_A - aX_A$, $G_2 = -X_A - aY_A$, $G_3 = ax_C - y_C$, $G_4 = x_C + ay_C$, $G_5 = a$,

$G_6 = c$, $G_7 = cz_C$, $G_8 = cZ_A$, $G_9 = d$

$X_i = X_{Di} \cos\alpha_{ji} + Y_{Di} \sin\alpha_{ji}$, $Y_i = -X_{Di} \sin\alpha_{ji} + Y_{Di} \cos\alpha_{ji}$

Based on stationary state condition $S = \sum [\Delta q_i]^2$

$\frac{dS}{dj} = 0$, $(j = G_1 \div G_9)$

considering Δ_{q_i} we obtain the following linear system:

$A \cdot \overline{G} = \overline{B}$

where, matrix elements A (9,9):

$a_{11} = \sum \cos^2 \alpha_{ji}$, $a_{12} = a_{21} = \frac{1}{2} \sum \sin\alpha_{ji} \cos\alpha_{ji}$, $a_{13} = a_{31} = \sum \cos\alpha_{ji} \cdot \cos(\alpha_{ji} - \beta_{ji})$,

$a_{14} = a_{41} = \sum \cos\alpha_{ji} \cdot \sin(\alpha_{ji} - \beta_{ji})$, $a_{15} = a_{51} = \sum X_i \cdot \cos\alpha_{ji}$, $a_{16} = a_{61} = \sum Z_i \cdot \cos\alpha_{ji}$,

$a_{17} = a_{71} = \sum \cos\alpha_{ji}$, $a_{18} = a_{81} = \sum \cos\alpha_{ji}$, $a_{19} = a_{91} = \sum \cos\alpha_{ji}$, $a_{22} = \sum \sin^2 \alpha_{ji}$,

$a_{23} = a_{32} = \sum \sin\alpha_{ji} \cdot \cos(\alpha_{ji} - \beta_{ji})$, $a_{24} = a_{42} = \sum \sin\alpha_{ji} \cdot \sin(\alpha_{ji} - \beta_{ji})$,

$a_{25} = a_{52} = \sum X_i \cdot \sin\alpha_{ji}$, $a_{26} = a_{62} = \sum Z_i \cdot \sin\alpha_{ji}$, $a_{27} = a_{72} = \sum \sin\alpha_{ji}$,

$a_{28} = a_{82} = \sum \sin\alpha_{ji}$, $a_{29} = a_{92} = \sum \sin\alpha_{ji}$, $a_{33} = \sum \cos^2(\alpha_{ji} - \beta_{ji})$,

$a_{34} = a_{43} = \frac{1}{2} \sum \sin(\alpha_{ji} - \beta_{ji}) \cdot \cos(\alpha_{ji} - \beta_{ji})$, $a_{35} = a_{53} = \sum X_i \cdot \sin(\alpha_{ji} - \beta_{ji})$,

$a_{36} = a_{63} = \sum Z_i \cdot \cos(\alpha_{ji} - \beta_{ji})$, $a_{37} = a_{73} = \sum \cos\alpha_{ji}(\alpha_{ji} - \beta_{ji})$, $a_{38} = a_{83} = \sum \cos(\alpha_{ji} - \beta_{ji})$

$a_{39} = a_{93} = \sum \cos(\alpha_{ji} - \beta_{ji})$, $a_{44} = \sum \sin^2(\alpha_{ji} - \beta_{ji})$,

$a_{45} = a_{54} = \sum X_i \cdot \sin(\alpha_{ji} - \beta_{ji})$, $a_{46} = a_{64} = \sum Z_i \cdot \sin(\alpha_{ji} - \beta_{ji})$

$a_{47} = a_{74} = \sum \sin(\alpha_{ji} - \beta_{ji})$, $a_{48} = a_{84} = \sum \sin(\alpha_{ji} - \beta_{ji})$, $a_{49} = a_{94} = \sum \sin(\alpha_{ji} - \beta_{ji})$,

$a_{55} = \sum X_i^2$, $a_{56} = a_{65} = \sum X_i Z_i$, $a_{57} = a_{75} = \sum X_i$, $a_{58} = a_{85} = \sum X_i$, $a_{59} = a_{95} = \sum X_i$,

$a_{66} = \sum Z_i^2$, $a_{67} = a_{76} = \sum Z_i$, $a_{68} = a_{86} = \sum Z_i$, $a_{69} = a_{96} = \sum Z_i$,

$a_{77} = a_{88} = a_{99} = N$, $a_{78} = a_{87} = a_{79} = a_{97} = a_{89} = a_{98} = 1$,

$X = [G_1, G_2, G_3, G_4, G_5, G_6, G_7, G_8, G_9]^T$,

$\overline{B} = [\sum Y_i \cos\alpha_{ji}, \sum Y_i \sin\alpha_{ji}, \sum Y_i \cos(\alpha_{ji} - \beta_{ji}), \sum Y_i \sin(\alpha_{ji} - \beta_{ji}),$

$\sum Y_i Y_i, \sum Y_i Z_i, \sum Y_i, \sum Y_i, \sum Y_i]^T$

System solution enables to determine the required parameters as:

$X_A = \frac{-G_2 - aG_1}{1 + a^2}$, $Y_A = \frac{-aG_2 + G_1}{1 + a^2}$, $Z_A = -\frac{G_8}{G_6}$, $x_C = \frac{aG_3 + G_4}{1 + a^2}$, $y_C = \frac{aG_4 - G_3}{1 + a^2}$, $z_C = \frac{G_7}{G_6}$, $b = G_5$, $c = G_6$, $d = G_9$.

As a consequence of solution of this problem, we determine such coordinates of points $A(X_A, Y_A, Z_A) \in Q$, $C(x_C, y_C, z_C) \in Q_2$, and plane on solid Q_1 that by combining plane and spherical kinematic pairs with them, we obtain open-link spatial kinematic chain ACD of type $RP_L S$ (R – rotational, P_L – plane, S - spherical kinematic pairs).

3. Let us assume that $c = -1$. Then weighted difference considering (4) is as follows

$$\Delta q_i = G_1 \cos\alpha_{ji} + G_2 \sin\alpha_{ji} + G_3 \cos(\alpha_{ji} - \beta_{ji}) + G_4 \sin(\alpha_{ji} - \beta_{ji}) +$$

$$+G_5X_i+G_6Y_i+G_7+G_8+G_9-Z_{Di}$$

where,

$G_i=-aX_A-вY_A$, $G_2=bX_A-aY_A$, $G_3=ax_C+вy_C$, $G_4=(ay_C-bx_C)$, $G_5=a$,

$G_6=b$, $G_7=z_C$, $G_8=Z_A$, $G_9=d$.

Based on stationary state condition $S=\sum[\Delta q_i]^2$

$$\frac{dS}{dj}=0,\ (j=G_1\div G_9)$$

considering Δ_{q_i} we obtain the following linear system:

$$A\cdot\overline{G}=\overline{B}$$

where, matrix elements A (9,9):

$a_{11}=\sum\cos^2\alpha_{ji}$, $a_{12}=a_{21}=\frac{1}{2}\sum\sin\alpha_{ji}\cos\alpha_{ji}$, $a_{13}=a_{31}=\sum\cos\alpha_{ji}\cdot\cos(\alpha_{ji}-\beta_{ji})$,

$a_{14}=a_{41}=\sum\cos\alpha_{ji}\cdot\sin(\alpha_{ji}-\beta_{ji})$, $a_{15}=a_{51}=\sum X_i\cdot\cos\alpha_{ji}$, $a_{16}=a_{61}=\sum Y_i\cdot\cos\alpha_{ji}$,

$a_{17}=a_{71}=\sum\cos\alpha_{ji}$, $a_{18}=a_{81}=\sum\cos\alpha_{ji}$, $a_{19}=a_{91}=\sum\cos\alpha_{ji}$, $a_{22}=\sum\sin^2\alpha_{ji}$,

$a_{23}=a_{32}=\sum\sin\alpha_{ji}\cdot\cos(\alpha_{ji}-\beta_{ji})$, $a_{24}=a_{42}=\sum\sin\alpha_{ji}\cdot\sin(\alpha_{ji}-\beta_{ji})$,

$a_{25}=a_{52}=\sum X_i\cdot\sin\alpha_{ji}$, $a_{26}=a_{62}=\sum Y_i\cdot\sin\alpha_{ji}$, $a_{27}=a_{72}=\sum\sin\alpha_{ji}$,

$a_{28}=a_{82}=\sum\sin\alpha_{ji}$, $a_{29}=a_{92}=\sum\sin\alpha_{ji}$, $a_{33}=\sum\cos^2(\alpha_{ji}-\beta_{ji})$,

$a_{34}=a_{43}=\frac{1}{2}\sum\sin(\alpha_{ji}-\beta_{ji})\cdot\cos(\alpha_{ji}-\beta_{ji})$, $a_{35}=a_{53}=\sum X_i\cdot\cos(\alpha_{ji}-\beta_{ji})$,

$a_{36}=a_{63}=\sum Y_i\cdot\cos(\alpha_{ji}-\beta_{ji})$, $a_{37}=a_{73}=\sum\cos\alpha_{ji}(\alpha_{ji}-\beta_{ji})$, $a_{38}=a_{83}=\sum\cos(\alpha_{ji}-\beta_{ji})$

$a_{39}=a_{93}=\sum\cos(\alpha_{ji}-\beta_{ji})$, $a_{44}=\sum\sin^2(\alpha_{ji}-\beta_{ji})$,

$a_{45}=a_{54}=\sum X_i\cdot\sin(\alpha_{ji}-\beta_{ji})$, $a_{46}=a_{64}=\sum Y_i\cdot\sin(\alpha_{ji}-\beta_{ji})$

$a_{47}=a_{74}=\sum\sin(\alpha_{ji}-\beta_{ji})$, $a_{48}=a_{84}=\sum\sin(\alpha_{ji}-\beta_{ji})$, $a_{49}=a_{94}=\sum\sin(\alpha_{ji}-\beta_{ji})$,

$a_{55}=\sum X_i^2$, $a_{56}=a_{65}=\sum X_iY_i$, $a_{57}=a_{75}=\sum X_i$, $a_{58}=a_{85}=\sum X_i$, $a_{59}=a_{95}=\sum X_i$,

$a_{66}=\sum Y_i^2$, $a_{67}=a_{76}=\sum Y_i$, $a_{68}=a_{86}=\sum Y_i$, $a_{69}=a_{96}=\sum Y_i$,

$a_{77}=a_{88}=a_{99}=N$, $a_{78}=a_{87}=a_{79}=a_{97}=a_{89}=a_{98}=1$,

$$X=[G_1,G_2,G_3,G_4,G_5,G_6,G_7,G_8,G_9]^T,$$

$$\overline{B}=[\sum Z_{Di}\cos\alpha_{ji},\sum Z_{Di}\sin\alpha_{ji},\sum Z_{Di}\cos(\alpha_{ji}-\beta_{ji}),\sum Z_{Di}\sin(\alpha_{ji}-\beta_{ji}),$$
$$\sum Z_{Di}X_i,\sum Z_{Di}Y_i,\sum Z_{Di},\sum Z_{Di},\sum Z_{Di}]^T$$

System solution enables to determine the required parameters as:

$$X_A=\frac{bG_2-aG_1}{a^2+b^2},\quad Y_A=-\frac{bG_1+aG_2}{a^2+b^2},\quad Z_A=G_8,\quad x_C=\frac{aG_3-bG_4}{a^2+b^2},$$

$$y_C=\frac{aG_4+bG_3}{a^2+b^2},\quad z_C=G_7\ \ a=G_5,\quad b=G_6,\quad d=G_9.$$

As a consequence of solution of this problem, we determine such coordinates of points $A(X_A,Y_A,Z_A)\in Q$, $C(x_C,y_C,z_C)\in Q_2$ and plane on solid Q_1 that by combining plane and spherical kinematic pairs with them, we obtain open-link spatial kinematic chain ACD of type RP_LS (R – rotational, P_L – plane, S - spherical kinematic pairs).

In the cases of normalization $d=-1$ the course of problem solution remains analogical.

By fixing or specifying a part of the required parameters of synthesis in various combinations, we may obtain different modifications of BKC with the plane and spherical pairs:

a) Let there be given coordinates X_{Ai}, Y_{Ai}, Z_{Ai} of point A, coordinates X_{Di}, Y_{Di}, Z_{Di} of point D and Euler angles $Q_1(\psi_{1i}, \theta_{1i}, \varphi_{1i})$, $Q_2(\psi_{2i}, \theta_{2i}, \varphi_{2i})$. Then as result of synthesis, we can obtain three-link spatial BKC ACD.

$$\Delta q_i = G_1 \cos(\alpha_{ji} - \beta_{ji}) + G_2 \sin(\alpha_{ji} - \beta_{ji}) + G_3 X_i + +G_4 Y_i + G_5 Z_i + G_6 - 1,$$

where,

$$G_1 = (ax_C + \text{в} y_C),\ G_2 = ay_C - bx_c,\ G_3 = a,\ G_4 = \text{в},\ G_5 = c,\ G_6 = cz_C.$$

$$\begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix} = \begin{bmatrix} \cos\alpha_{ji} & \sin\alpha_{ji} & 0 \\ -\sin\alpha_{ji} & \cos\alpha_{ji} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} X_{Di} - X_{Ai} \\ Y_{Di} - Y_{Ai} \\ Z_{Di} - Z_{Ai} \end{bmatrix}$$

Let us comprise sum of squares of weighted difference for N positions

$$S = \sum_{i=1}^{N} [\Delta q_i]^2 \quad (i = 1, N)$$

Stationary conditions per variables

$$\frac{\partial S}{\partial j} = 0 \quad (j = G_1, G_2, G_3, G_4, G_5, G_6);$$

would result in the following simultaneous linear algebraic equations as $G_1 \div G_6$;

$$A \cdot \overline{G} = \overline{B}$$

where, matrix elements A (6,6):

$$a_{11} = \sum \cos^2(\alpha_{ji} - \beta_{ji}),\quad a_{12} = a_{21} = \frac{1}{2}\sum \sin(\alpha_{ji} - \beta_{ji}) \cdot \cos(\alpha_{ji} - \beta_{ji}),\quad a_{13} = a_{31} = \sum X_i \cos(\alpha_{ji} - \beta_{ji}),$$

$$a_{14} = a_{41} = \sum Y_i \cos(\alpha_{ji} - \beta_{ji})$$

$$a_{15} = a_{51} = \sum Z_i \cos(\alpha_{ji} - \beta_{ji}),\ a_{16} = a_{61} = \sum \cos(\alpha_{ji} - \beta_{ji})$$

$$a_{22} = \sum \sin^2(\alpha_{ji} - \beta_{ji}),\ a_{23} = a_{32} = \sum X_i \sin(\alpha_{ji} - \beta_{ji}),$$

$$a_{24} = a_{42} = \sum Y_i \sin(\alpha_{ji} - \beta_{ji}),\ a_{25} = a_{52} = \sum Z_i \sin(\alpha_{ji} - \beta_{ji}),$$

$$a_{26} = a_{62} = \sum \sin(\alpha_{ji} - \beta_{ji}),\quad a_{33} = \sum X_i^2,\quad a_{34} = a_{43} = \sum X_i Y_i \quad a_{35} = a_{53} = \sum X_i Z_i,\quad a_{36} = a_{63} = \sum X_i,$$

$$a_{44} = \sum Y_i^2,\ a_{45} = a_{54} = \sum Y_i Z_i,\ a_{46} = a_{64} = \sum Y_i$$

$$a_{55} = \sum Z_i^2,\ a_{56} = a_{65} = \sum Z_i,\ a_{66} = N,$$

$$X = [G_1, G_2, G_3, G_4, G_5, G_6, G_7, G_8, G_9]^T,$$

$$\overline{B} = \left[\sum \cos(\alpha_{ji} - \beta_{ji}), \sum \sin(\alpha_{ji} - \beta_{ji}), \sum X_i, \sum Y_i, \sum Z_i, N\right]^T$$

System solution enables you to determine the required parameters of synthesis as:

$$x_C = \frac{aG_1 - bG_2}{a^2 + b^2},\ y_C = \frac{bG_1 + aG_2}{a^2 + b^2},\ z_C = \frac{G_6}{G_5},\ a = G_3,\ b = G_4\ \ c = G_5,\ a^2 + b^2 \neq 0.$$

б) If Euler angles $\psi_{1i} = \theta_{1i} = \varphi_{1i} = 0$ of solid Q_1, then a problem reduces to the weighted square approximation of point $C_i(x_C, y_C, z_C)$ with N approximate-collinear positions, etc.

Example: Let us consider problem of synthesis of spatial four-link mechanism of type RP_LS (R – rotational, P_L – plane, S - spherical kinematic pairs). Approximately reproducing function $\psi = -50\cos\frac{6}{5}\varphi$, $\varphi \in [0^0, 120^0]$. We divide an interval $[0^0, 120^0]$ into 20 equal parts (Figure 2). If we are given axial angle $\rho = 90^\circ$ and coordinates $X_D = Z_D = 0$, $Y_D = 1{,}2$ of point D, let us determine the following parameters of four-link chain $ABCD$:

$$a = \sqrt{x_B^2 + y_B^2 + z_B^2},\ b = R,\ c = \sqrt{x_C^2 + y_C^2 + z_C^2},\ X_A,\ Y_A,\ Z_A,\ \varphi_0 = \arccos\frac{z_B}{a},\ \psi_0 = \arccos\frac{z_C}{c}.$$

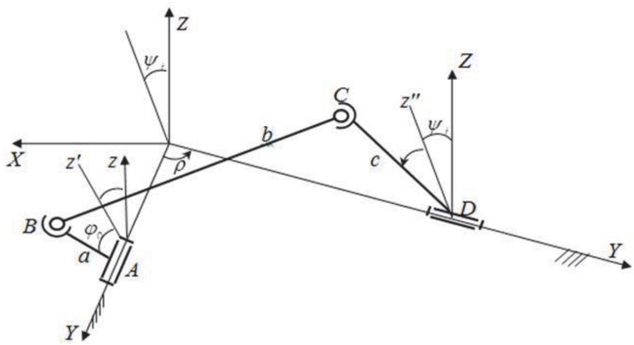

Fig. 2 Four link spatial chain ABCD.

For synthesis of this problem, we use expression (2). Then matrix is as follows:

$$T_{jk}^{i} = T_{01} \cdot T_{02} \cdot T_{03},$$

where,

$$T_{01} = \begin{bmatrix} \cos\varphi_i & 0 & -\sin\varphi_i \\ 0 & 1 & 0 \\ \sin\varphi_i & 0 & \cos\varphi_i \end{bmatrix}, \quad T_{02} = \begin{bmatrix} \cos\rho & -\sin\rho & 0 \\ \sin\rho & \cos\rho & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad T_{03} = \begin{bmatrix} \cos\psi_i & 0 & \sin\psi_i \\ 0 & 1 & 0 \\ -\sin\psi_i & 0 & \cos\psi_i \end{bmatrix}$$

On order to determine the required parameters of synthesis, we use search algorithm of minimum of sum S. Based on above minimization algorithm, the value of minimum of sum $S = 0.00012$.

Then a problem of mechanism synthesis reduces to minimization of the objective function of BKC

$$S = \sum_{i=1}^{21} [\Delta_{qi}(X_A, Y_A, Z_A, x_B, y_B, z_B, R, x_C, y_C, z_C)]^2$$

For solution of this problem we apply the above given search algorithm of minimum of sum S. According to algorithm, we are given 10 values $B(x_B^{(0)}, y_B^{(0)}, z_B^{(0)})$ и $C(x_C^{(0)}, y_C^{(0)}, z_C^{(0)})$ depending on the length of links a and c. For each preliminary value of points $B(x_B^{(0)}, y_B^{(0)}, z_B^{(0)})$ and $C(x_C^{(0)}, y_C^{(0)}, z_C^{(0)})$ in Table 1 are given the results of calculations.

Iteration process of minimum search of function S is completed upon satisfaction of inequation

$$|R^{(k)} - R^{(k-1)}| \le \varepsilon, \quad |X_A^{(k)} - X_A^{(k-1)}| \le \varepsilon, \quad |Y_A^{(k)} - Y_A^{(k-1)}| \le \varepsilon, \quad |Z_A^{(k)} - Z_A^{(k-1)}| \le \varepsilon,$$ where, $\varepsilon = 10^{-4}$.

3. Results

Table 1: Figures 3 – 6 demonstrate $2D$ and $3D$ graphics of objective function S.

N	x_B	y_B	z_B	R	x_C	y_C	z_C	X_A	Y_A	Z_A	S
1	1,014	1,815	1,2968	2,6406	0,804	1,9819	0,3277	0,0001	1,204	0	0,00152
2	0,9178	1,7169	0,8219	2,41518	0,9017	1,7618	0,3838	0,0002	1,105	0	0,00103
3	-1,1171	1,6449	0,7263	1,9678	0,7664	0,9261	0,4147	0	1,086	0,0001	0,00122
4	-0,3181	1,4716	0,9227	1,7646	0,5464	0,8167	0,4386	0	1,2667	0,0001	0,00052
5	-0,22763	1,21314	0,609801	1,69127	0,652906	0,639147	0,469694	0	1,189776	0	0,00012
6	0,7819	1,0449	0,3233	1,5118	0,5917	0,6221	0,4554	0	1,0997	0,0001	0,00018
7	0,1918	1,1667	0,2218	1,7246	0,4928	0,6046	0,7184	0,0003	1,6756	0	0,00021
8	0,2128	1,0717	0,3486	1,54714	0,5154	0,7654	0,6656	0,0002	1,7617	0	0,00146
9	0,56179	1,00818	0,9417	1,3349	0,3426	0,6406	0,6846	0,0001	1,3606	0,0002	0,0019
10	0,86718	1,2617	0,6461	1,2667	0,5617	0,7627	0,6976	0,0001	1,1415	0,000	0,00031

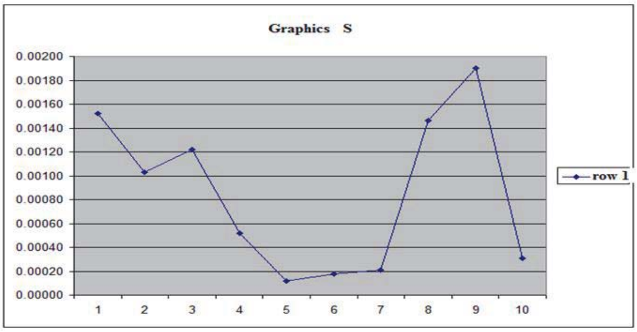

Fig .3 Number of points S .

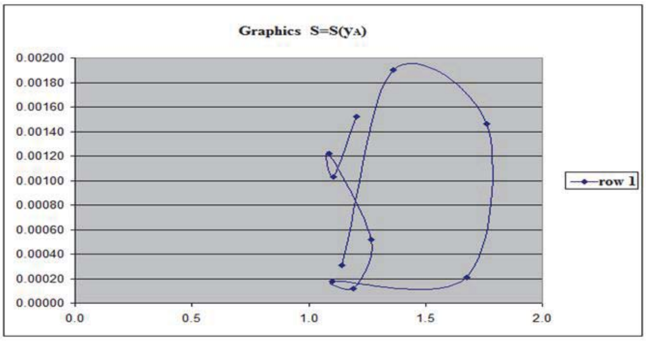

Fig .4 Number of function points $S = S(Y_A)$.

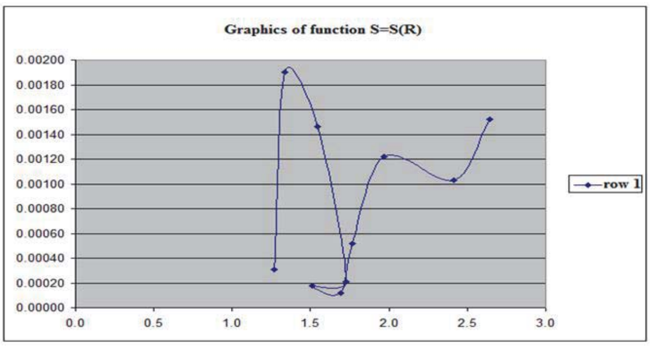

Fig .5 Number of function points $S = S(R)$.

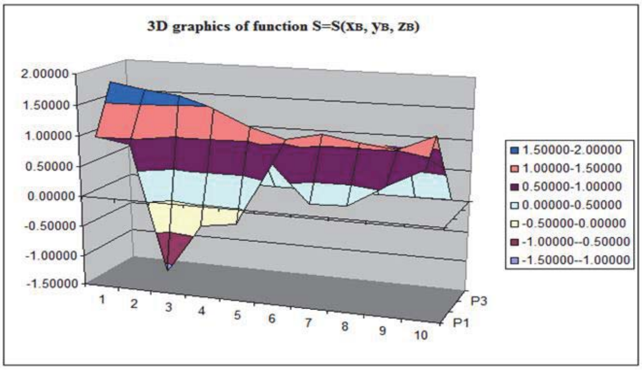

Fig .6 3D graphics of function $S = S(x_B, y_B, z_B)$.

Values of required parameters of synthesized mechanism:
$x_B = -0{,}227631$, $x_C = 0{,}652906$, $y_B = 1{,}21314$, $y_C = 0{,}639147$, $z_B = 0{,}609801$, $z_C = 0{,}469694$, $X_A = Z_A = 0$, $Y_A = 1{,}189776$, $R = 1{,}69127$

4. Discussion

Thus, the use of one and the same objective function being generated for the synthesis of BKC and its modification, allows automating the process of synthesis of spatial linkage mechanisms as per predetermined positions of input and output links of the mechanism.

5. Conclusion

In summary, when there is a synthesis of BKC with spherical kinematic pairs as per predetermined positions of the input and output links of the mechanism, and when two adjacent links of BKC are tending to infinity, it is necessary to replace the spherical kinematic pair for plain or cylindrical. In this case, the synthesized mechanism takes a form of spatial link mechanism after determining the required parameters.

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